The Final exam will have problems of the same type as these but also may have a problem or two similar to those given in review lectures. About a third of the final will be similar to homework problems after exam 2, see 10 below.

All the answers I have are on the last page. If you wish to contribute an answer, send it to me at (miller@math.wisc.edu). Bare-bones answers only, please do not give out completely worked solutions. Completely worked out solutions convert an exercise into an example, and while examples are useful learning tools, exercises are vastly more important.

1. $\lim_{x\to 2}(2x+3)=7$ Show that this is true using an $\epsilon-\delta$ argument.

1a. same for $\lim_{x\to 3}(x^2-2)$

1b. same for $\lim_{x\to 1}\frac{x}{1+x}$

2. $\frac{d}{dx}(3x^2+2)=6x$ Show that this is true directly from the definition of derivative and the limit rules. No points will be given for quoting rules of differentiation.

2a. same for $\frac{d}{dx}\sqrt{1+x^2}$

2b. same for $\frac{d}{dx}\frac{1}{1-2x}$

3. Find the derivative of

$$f(x)=\frac{\tan(x)}{1+x^2}$$

3a. same for $\ln(x^2+e^x)$

3b. same for $\sin(\ln(x))$

4. For what values of a and b is it true that f is a differentiable function?

$$f(x)=\begin{cases} ax & \text{if } x\le 1\\ x^2+b & \text{if } x>1\end{cases}$$

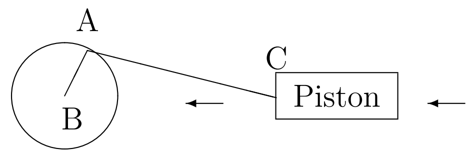
4a. same for

$$f(x) = \begin{cases} ax & \text{if } x \leq -1 \\ x^2 + b & \text{if } x > -1 \end{cases}$$

4b. same for

$$f(x) = \begin{cases} e^x & \text{if } x \leq 0 \\ ax + b & \text{if } x > 0 \end{cases}$$

5. A piston is being driven back and forth by a crankshaft which is turning (see diagram). The points A,B,C are swivel joints. The point B which is the axis of the crankshaft is held fixed while the point A rotates around counterclockwise exactly once per unit of time. The point C on the piston moves horizontally back and forth. The two connecting arms AB and AC are rigid and of length 1 unit and 3 units respectively. Find the speed the piston (point C) is moving when the point A is directly above the point B. What is the speed when the angle ABC is 45 degrees?



5a. A particle is constrained to move along a parabola whose equations is $y = x^2$. At what point on the curve are the x and y coordinates changing at the same rate?

5b. A 6 foot man walks away from a 10 foot high lamp at the rate of 3 ft per sec. How fast is the tip of his shadow moving?

6. You are designing a mural and you would like to have a margin of 5 feet on the left, right, and top of the artwork, but none on the bottom. If you allow 800 ft^2 for the area containing the artwork itself, what dimensions should the wall have if you want to minimize the total area of the wall (i.e., of the artwork and the margins.)

6a. A man has 112 miles of fencing for inclosing two separate lots, one of which is to be a square and the other a rectangle which is three times as long as it is wide. Find the dimensions of each lot so that the total area which is inclosed shall be a minimum.

6b. The intensity of illumination at any point is proportional to the product of the strength of the light source and the inverse square distance from the source. If two sources of strengths a and b are at a distance c apart, at what point on the line joining them will the intensity be a minimum? Assume the intensity at any point is the sum of intensities from the two sources.

6c. Find the area of the largest rectangle with lower base on the $x-$axis and upper vertices on the curve $y = 9 - x^2$.

6d. Two poles, one 1 meter tall and one 2 meters tall, are 4 meters apart. A telephone wire is attached to the top of each pole and it is also staked to the ground somewhere between the two poles. Where should the wire be staked so that the angle formed by the two pieces of wire at the stake is a maximum?

7. Recall the $\epsilon - \delta$ definition of the Riemann integral: $\int_a^b f(x)dx = I$ iff for every $\epsilon > 0$, there exists a $\delta > 0$ such that for any partition of $[a, b]$, $a = x_0 < x_1 < \cdots < x_n = b$ with $x_k - x_{k-1} = \Delta x_k \leq \delta$ and sample points c_k with $x_{k-1} \leq c_k \leq x_k$ for $k = 1 \ldots n$ we have

$$\left| f(c_1)\Delta x_1 + f(c_2)\Delta x_2 + \cdots + f(c_n)\Delta x_n - I \right| \leq \epsilon$$

Define

$$f(x) = \begin{cases} 0 & \text{if } x = 1 \\ -3 & \text{if } x \neq 1 \end{cases}$$

(a) Draw the graph of f. What is $I = \int_0^2 f(x)dx$?

(b) Using the $\epsilon - \delta$ definition of the integral show that f is Riemann integrable, i.e., given $\epsilon > 0$ what should $\delta > 0$ be? Explain why a larger δ will not work.

7a. same for

$$f(x) = \begin{cases} 0 & \text{if } x \le 1 \\ 4 & \text{if } x > 1 \end{cases}$$

8. Find the limit or show that it is not defined:

8(a)

$$\lim_{x \to 1} \frac{2\sqrt{x} - 2x}{1 - x}$$

8(b)

$$\lim_{x \to 1} \frac{2\sqrt{x} - x}{(1 - x)^2}$$

8(c)

$$\lim_{x \to 1} \frac{2\sqrt{x} - x}{1 - x}$$

8(d)

$$\lim_{x \to \infty} \sqrt{x+1} - \sqrt{x}$$

8(e)

$$\lim_{x \to \infty} (x+1)^2 - x^2$$

9. Find intervals where it is increasing or decreasing. Find local maxima and minima. Find inflection points and intervals where it is concave up or down. Find limits at ∞ and 0^+. Sketch graph.

$$f(x) = \frac{\ln(x)}{x}$$

9a. same for $f(x) = \frac{e^x}{x}$ on $(0, \infty)$

9b. same for $f(x) = \frac{x^2-1}{x}$ on $(0, \infty)$

9c. same for $f(x) = \sin(\frac{1}{x})$ on $(0, \infty)$

10. 8.02 : 2 4 6 11 12 13
8.06 : 2 3 4 7 9 11
Extra : 39 40
8.09 : 1 2 3 4 5 6

Answers

4. $a = 2$ and $b = 1$

5. -2π and $-\pi(\sqrt{2} + \frac{1}{\sqrt{8.5}})$.

5. a. $(\frac{1}{2}, \frac{1}{4})$

5. b. $\frac{15}{2}$ ft per sec.

6. Artwork 40 feet wide and 20 feet high.

6. a. 12 x 12 and 8 x 24

6. b. $c(\frac{a^{1/3}}{a^{1/3}+b^{1/3}})$ units from the source of strength a.

6. c. $12\sqrt{3}$