

MATH 221

SECOND EXAM

PASSMAN

NAME \_\_\_\_\_

T. A.'s NAME \_\_\_\_\_

Do ALL 7 problems and show ALL work.  
Each problem is worth 20 points.  
Use only techniques that have been covered in class.

PROBLEM	GRADE
1 20 pts	
2 20 pts	
3 20 pts	
4 20 pts	
5 20 pts	
6 20 pts	
7 20 pts	
TOTAL	

1. a) Find the absolute maximum and minimum of the function

$$y = x(4 - x)^3$$

in the closed interval  $[0, 3]$ .

b) Find the tangent line to the curve

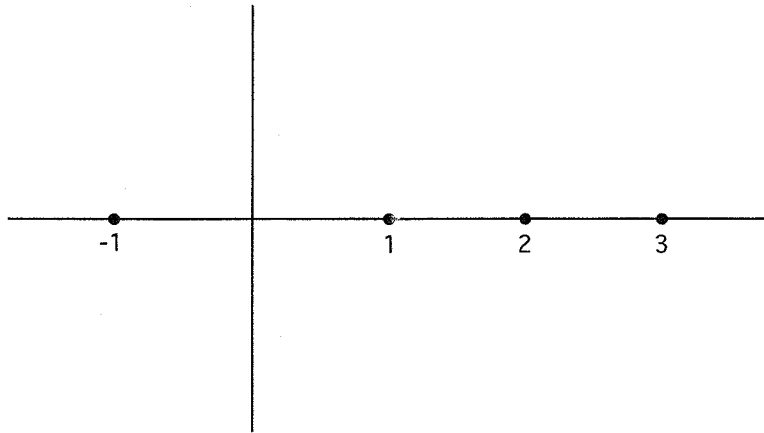
$$x^2 + 3xy - y^2 = 3$$

at the point  $(1, 2)$ .

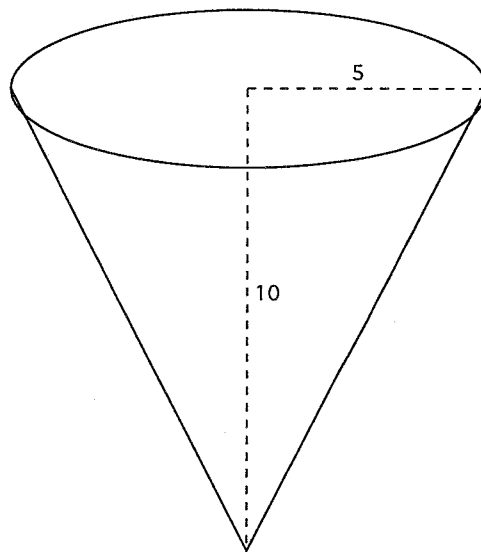
2. Determine where the curve

$$y = x^2(x - 3)$$

is increasing or decreasing, concave up or concave down. Where are its critical points, local maxima and minima, and inflection points? Use this information to sketch the curve on the axes below.



3. A tank shaped like a circular cone of height 10 and radius 5 is being filled from the top with water at a constant rate of  $dV/dt = 4\pi$ . How fast is the water level increasing when the height of the water is equal to 4.



4. Find the point  $P(x, y)$  on the curve  $y = 2\sqrt{x}$  (with  $x \geq 0$ ) that is closest to the point  $Q(2, 3\sqrt{3})$ .

5. Evaluate the indefinite integrals

a)

$$\int (\sec x + \cos x) \tan x \, dx$$

b)

$$\int 2x(2x^{1/3} + 7)^2 \, dx$$

6. Estimate  $\int_0^2 x^2 (2 - x) dx$  by partitioning the interval  $[0, 2]$  into four equal length pieces and by using the midpoint of each subinterval as the random point in the Riemann sum. Write your answer as a fraction. Compare this to the precise value of the integral by writing both of these fractions over a common denominator.

7. a) Let

$$f(x) = \int_{x^4}^1 \frac{dt}{1 + \sqrt{t}}.$$

Find  $f(1)$  and  $df/dx$ .

b) Find the area between the curve  $y = x(x - 1)(x - 2)$  and the  $x$ -axis in the interval  $[0, 2]$ .