CALCULUS 221

11th WEEK EXAM

I. M. Isaacs
Thursday, November 13, 2008
5:30 – 7:00 P.M.

Do all problems — 100 points.
Use backs of pages for scrap, or if you need more space.

NAME: _____

TA: _____

Do not write below here.

Prob. 1: _____ out of 21.

Prob. 2: _____ out of 21.

Prob. 3: _____ out of 11.

Prob. 4: _____ out of 11.

Prob. 5: _____ out of 12.

Prob. 6: _____ out of 7.

Prob. 7: _____ out of 7.

Prob. 8: _____ out of 10.

Total: _____ out of 100.

1. [21 POINTS] Antidifferentiate.

(a) $\int x^2(6-5x^2)\,dx$

(b) $\int \frac{\sin(1/z)}{z^2}\,dz$

(c) $\int \sec^5(2\theta)\tan(2\theta)\,d\theta$

2. [21 POINTS] Compute these.

(a) $\displaystyle\int_1^2 \frac{2x^3+1}{x^2}\,dx$

(b) $\displaystyle\int_0^{\pi/6} \sec^2(2\theta)\,d\theta$

(c) $\displaystyle\int_{-1}^{3} \frac{(2t+1)\,dt}{\sqrt{t^2+t+4}}$

3. [11 POINTS] Let $f(x) = \begin{cases} 2x^4 + x & \text{if } x \leq 1 \\ 4 - x & \text{if } x > 1 \,. \end{cases}$

Find the maximum and minimum values of $f(x)$ on the interval $[-1, 2]$, and find all points where these occur. If there is no maximum or there is no minimum, say so, and explain **briefly** how you know.

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4. [11 POINTS] The graph of $y = g(x)$ crosses the y-axis at $y = 1$, and at each point (x, y) on the curve, the slope of the tangent line is $-2xy^2$. Compute $g(1)$.

5. [12 POINTS] I want to compute a decent approximation to $\sqrt[3]{7}$.

(a) First, I try a linear approximation to the function $y = x^{1/3}$ near the point $(8, 2)$. Find the approximation to $\sqrt[3]{7}$ that I obtain this way.

(b) I decide that the linear approximation is probably not close enough, and that I had better use Newton's method to approximate a solution of the equation $x^3 = 7$. I start with a first guess $x_0 = 2$. In practice, I would apply Newton's method several times, but if I apply it just once, what is the next guess that I get? (In other words, find x_1.)

6. [7 POINTS] it is easy to see that $\int_0^2 3x^2 = 8$.

Compute the Riemann sum corresponding to this definite integral, where the interval is partitioned into four equal parts and I choose the right endpoint of each subinterval as my sample point.

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7. [7 POINTS] Compute $\lim_{x \to 1} \frac{2\sqrt{x} - x}{(1-x)^2}$. If the limit does not exist, say so.

8. [10 POINTS] A few hours after a spill of 10 cubic meters of oil on the surface of a lake, the oil covers a circular patch of water 100m in radius, and the thickness of the oil is uniform. (In other words, the thickness is the same everywhere.) If the radius of the oil slick is increasing at 10 m/hr, how fast is the thickness of the oil changing?

THE END