Instructions

No notes, no books, no calculators, no cell phones, no pagers, no electronic devices of any kind.

Show all of your work. Circle your answer.

Name_____

Circle your section number.
Hand in to your TA.

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Section
Number
 421 Cheng, Jingrui   07:45
 422 Cheng, Jingrui   08:50
 424 Lynch, John      09:55
 429 Lynch, John      01:20
 426 Powers, Michael  11:00
 427 Powers, Michael  12:05
 430 Soule, Michelle  01:20
 431 Soule, Michelle  02:25
 432 Charles, Zachary 07:45
 433 Charles, Zachary 02:25
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Problem	Points	Score
1	12	
2	12	
3	12	
4	12	
5	12	
6	12	
7	12	
8	14	
9	14	
10	12	
11	13	
12	13	
Total	150	

Solutions will be posted shortly after the exam:
www.math.wisc.edu/∼miller

1. (12 pts) Find the area of the bounded region between $y = 1 - x^2$ and $y = x + 1$.

2. (12 pts) Find the length of this curve:

$$x = 2t^3 + 1 \qquad y = 3t^2 \qquad 0 \leq t \leq 1$$

3. (12 pts)

$$\lim_{x \to 2} 3x - 1 \;=\; 5$$

Show that this is true using an $\epsilon - \delta$ argument.

4. (12 pts)

$$\frac{d}{dx}(2x^2 + 1) = 4x$$

Show that this is true directly from the definition of derivative. No points will be given for quoting rules of differentiation.

5. (12 pts) Find the derivatives of

$$(a) \qquad f(x) = \tan(1 + x^2)$$

$$(b) \qquad g(x) = 2x^3 + e^{\sin(x)}$$

6. (12 pts) For what values of a and b is it true that f is a differentiable function?

$$f(x) = \begin{cases} 2x + 1 & \text{if } x \leq 1 \\ ax^2 + b & \text{if } x > 1 \end{cases}$$

7. (12 pts) A particle is constrained to move along the graph of the equation

$$y = x^3 + x^2 + x + 1$$

For what values of x are the x and y coordinates necessarily changing at the same rate?

8. (14 pts)

$$f(x) = \frac{\ln(x)}{x}$$

Find intervals where it is increasing or decreasing. Find local maxima and minima. Find inflection points and intervals where it is concave up or down. Find limits at ∞ and 0^+. Sketch the graph.

9. (14 pts) You are designing a mural and you would like to have a margin of 1 feet on the left, right, and top of the artwork, but none on the bottom. If you allow 32 ft^2 for the area containing the artwork itself, what dimensions should the artwork have if you want to minimize the total area of the wall (i.e., of the artwork and the margins.)

10. (12 pts) Find the limit or show that it is not defined:

$$(a) \qquad \lim_{x \to 1} \frac{1 - x}{\sqrt{x} - x}$$

$$(b) \qquad \lim_{x \to \infty} \left(\sqrt{x^2 + x} \; - \; x\right)$$

11. (13 pts) R is the region below the curve $y = \frac{1}{\sqrt{1+x^2}}$ between $x = 0$, $x = 1$, and above the x-axis. Equivalently

$$R = \{(x, y) \ : 0 \leq x \leq 1 \text{ and } 0 \leq y \leq \frac{1}{\sqrt{1+x^2}}\}$$

Find the volume of the solid obtained by rotating R around the x-axis.

12. (13 pts) For the same R region as in the previous problem:

$$R = \{(x, y) \ : 0 \leq x \leq 1 \text{ and } 0 \leq y \leq \frac{1}{\sqrt{1+x^2}}\}$$

Find the volume of the solid obtained by rotating R around the y-axis.

Answers

1. $\frac{1}{6}$

2. $2(2^{\frac{3}{2}} - 1)$ or $4\sqrt{2} - 2$.

3. $|(3x - 1) - 5| = 3|x - 2| \leq 3\delta = \epsilon$ if $|x - 2| \leq \delta$ and $\delta = \frac{\epsilon}{3}$.

4. $\frac{f(x)-f(a)}{x-a} \to f'(a)$ as $x \to a$.

$$\frac{(2x^2 + 1) - (2a^2 + 1)}{x - a} = \frac{2(x^2 - a^2)}{x - a} = \frac{2(x - a)(x + a)}{x - a} = 2(x + a) \to 4a \text{ as } x \to a$$

5. (a) $(\sec(1 + x^2))^2 (2x)$ (b) $6x^2 + e^{\sin(x)} \cos(x)$

6. $a = 1$ and $b = 2$

7. $x = 0$ and $x = -\frac{2}{3}$.

8. Critical point at $x = e$, inflection point at $x = e^{\frac{3}{2}}$.

9. Artwork 8 feet wide and 4 feet high.

10. (a) 2 (b) $\frac{1}{2}$

11. $\frac{\pi^2}{4}$

12. $2\pi(\sqrt{2} - 1)$