

Instructions

No notes, no books, no calculators, no cell phones, no pagers, no electronic devices of any kind.

Show all of your work. Circle your answer.

Name _____

Circle your section number.

Hand in to your TA.

Section Number	Start Time
421 Cheng, Jingrui	07:45
422 Cheng, Jingrui	08:50
424 Lynch, John	09:55
429 Lynch, John	01:20
426 Powers, Michael	11:00
427 Powers, Michael	12:05
430 Soule, Michelle	01:20
431 Soule, Michelle	02:25
432 Charles, Zachary	07:45
433 Charles, Zachary	02:25

Problem	Points	Score
1	8	
2	8	
3	8	
4	8	
5	8	
6	8	
7	8	
8	8	
9	8	
10	8	
11	8	
12	6	
13	2	
14	2	
15	2	
Total	100	

Solutions will be posted shortly after the exam:

www.math.wisc.edu/~miller

1. (8 pts) State and prove the product rule of differentiation. Use complete sentences and correct grammar. Assume that the reader is an intelligent person who knows the rules of limits and the definition of the derivative but nothing more.

2. (8 pts) Give a direct proof from the definition of derivative that

$$\frac{d}{dx} \left(\frac{1}{x} \right) = -\frac{1}{x^2}$$

No points will be given for quoting the power rule.

3. (8 pts) A function $y = f(x)$ which satisfies the property that for every x that $y = f(x)$ is the minimum value y satisfying the equation

$$y^2 - xy + (x - 1) = 0$$

Give an explicit definition of f , i.e., a formula for f .

4. (8 pts)

Find numbers $a < b$ such that the function

$$f(x) = \sin(2x)$$

with domain restricted to $a \leq x \leq b$ has an inverse and $b - a$ is as big as possible. Give a formula for inverse function for f , $f^{-1}(x)$, using the arcsine function.

5. (8 pts) Use the $\epsilon - \delta$ definition to prove the following limit:

$$\lim_{x \rightarrow 9} \sqrt{x} = 3$$

6. (8 pts) Find

$$\lim_{t \rightarrow 0} \frac{3t}{\sin(2t)}$$

7. (8 pts) Find the following:

$$\lim_{t \rightarrow \infty} \frac{(At^3 + 4)^2}{(At^2 - 1)^3}$$

A is a constant which may be zero.

8. (8 pts) For what values of a and b is it true that f is a continuous function?

$$f(x) = \begin{cases} x + a & \text{if } x \leq 1 \\ x^2 + b & \text{if } 1 < x \leq 2 \\ x + 3 & \text{if } 2 < x \end{cases}$$

9. (8 pts) Find the first and second derivative of

$$f(x) = \sin^2(x)$$

10. (8 pts) For $p = \cos(\cos(\cos \theta))$ find $\frac{dp}{d\theta}$.

11. (8 pts) A crystal in the shape of a cube dissolves in acid so that the volume of the cube decreases by 5 cubic centimeters per minute. How fast is the edge of the cube changing when the volume is 1000 cubic centimeters?

12. (6 pts) Find the derivative of

$$f(x) = \sqrt{\frac{1+x}{2-x}}$$

13. (2 pts) A bottle is being filled with soda. The volume in ounces of soda at time t in seconds is $B(t)$. What units does the derivative $B'(t)$ have?

- (a) ounces
- (b) ounces per second
- (c) seconds
- (d) seconds per ounce
- (e) bottles of soda

14. (2 pts) Find $\lim_{x \rightarrow -3} (2x + 5)^{2013}$

- (a) 2013
- (b) $(-3)^{2013}$
- (c) ∞
- (d) -1
- (e) $-\infty$
- (f) The limit is undefined.

15. (2 pts) Suppose x is the repeating decimal

$$.47474747 \dots$$

Find integers p and q such that $x = \frac{p}{q}$.

- (a) $\frac{47}{100}$
- (b) $\frac{99}{47}$
- (c) $\frac{4}{7}$
- (d) $\frac{47}{99}$
- (e) This is impossible.

Answers

3. $y = \min(1, x - 1)$

4. $a = -\frac{\pi}{4}$, $b = \frac{\pi}{4}$, $f^{-1}(x) = \frac{1}{2} \arcsin(x)$

5. $\delta = \min(\epsilon, 1)$ works. The one is to insure we don't take the square root of a negative number.

$$|\sqrt{x} - 3| = \left| \frac{(\sqrt{x} - 3)(\sqrt{x} + 3)}{\sqrt{x} + 3} \right| = \frac{|x - 9|}{|\sqrt{x} + 3|} \leq \frac{|x - 9|}{3} \leq \delta \leq \epsilon$$

6. $\frac{3}{2}$

7. If $A = 0$, then the limit is -16 . If $A \neq 0$ then the limit is $\frac{1}{A}$.

8. $a = b = 1$

9. second derivative is $2 \cos^2(x) - 2 \sin^2(x)$.

10. $-\sin(\cos(\cos(\theta))) \sin(\cos(\theta)) \sin(\theta)$

11. $-\frac{1}{60}$ centimeters per minute or decreasing by $\frac{1}{60}$ centimeters per minute.

12. $-\frac{1}{2} \sqrt{\frac{2-x}{3}}$