

Angenent

math 221 — the second midterm — lectures 3 & 4  
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your ta: (circle one)

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Your name \_\_\_\_\_

1	(a)	(b)	(c)	(d)	(e)	
2						
3	(a)	(b)	(c)			
4	(a)	(b)	(c)	(d)	(e)	
5						
	<b>TOTAL</b>					

1. (30 pts) Compute these derivatives

(a)  $f(x) = \frac{2+x^2}{(1+x)^2} \implies f'(x) = \dots$

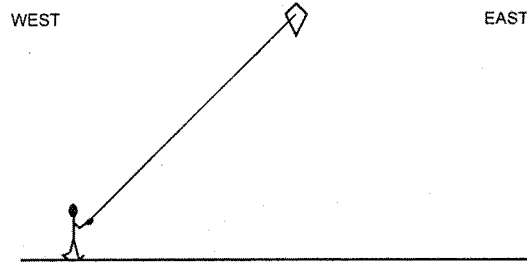
(b)  $\frac{d(\arcsin \sqrt{x})}{dx} = \dots$

(c)  $\frac{d \ln(2e^{\cos x})}{dx} = \dots$

(d)  $g(x) = \sqrt[4]{e^{2x} + \cos x} \implies g'(x) = \dots$

(e) Compute the 10<sup>th</sup> derivative of  $f(x) = \sin(2x) + \frac{1}{1-x}$ .  
(Use back of previous page for your solution)

2. (15 pts) I am flying a kite in a wind blowing toward the east. The kite is moving horizontally at an altitude of 400 feet, and so I must pay out string from the spool that I am holding. Find the rate at which the string is unwinding from the spool when the kite is directly over the point 300 feet east of where I am standing, if at that moment the kite is moving east at 10 feet per second (and staying at the same altitude all the time.)



3. (15 pts) A function  $y = f(x)$  satisfies the equation

$$2x + \sin(x) = 3y + y^3.$$

(a) Which of the following statements are true (you must of course explain your answer):

$$f(0) = 0?$$

$$f(\pi) = 2?$$

(b) Compute  $f'(0)$ .

(c) Can  $f'(x)$  ever be negative? Explain your answer.

4. (25 pts) Consider the function  $f(x) = \frac{x^2}{4+x^2}$ .

(a) Find the intervals on which  $f$  is increasing or decreasing.

(b) Find the local maxima and minima of  $f$ . Which of these are global maxima or minima?

(c) Find the inflection points on the graph of  $f$ , and the intervals where the function is convex and concave.

THIS PROBLEM CONTINUES ON THE NEXT PAGE

(d) Find all horizontal and vertical asymptotes of the graph of  $f$ .

(e) Sketch the graph.

5. (15 pts) You are designing a mural and you would like to have a margin of 5 feet on the left, right and top of the artwork, but none on the bottom. If you allow  $800 \text{ ft}^2$  for the area containing the artwork itself, what dimensions should the wall have if we want to minimize the total area of the wall (i.e. of the artwork and the margins.)

(First make a drawing!)