

MATH 221

FIRST EXAM

PASSMAN

NAME \_\_\_\_\_

T. A.'s NAME \_\_\_\_\_

Do ALL 7 problems and show ALL work.  
Each problem is worth 20 points.  
Use only techniques that have been covered in class.

PROBLEM	GRADE
1 20 pts	
2 20 pts	
3 20 pts	
4 20 pts	
5 20 pts	
6 20 pts	
7 20 pts	
TOTAL	

1. a) Find the equation of the line passing through the point  $(4, 1)$  and perpendicular to  $y = 2x + 19$ . At what point does the line you found cross the  $x$ -axis.

b) We are given two circles. The first is centered at the origin and has radius 15. The second is centered at  $(0, 4)$  and has radius 13. Find the points where these circles meet, that is their points of intersection.

2. a) The differentiable function  $y = f(x)$  satisfies  $f'(2) = 3$ . Find  $f(2)$  if the tangent line to the curve at  $(2, f(2))$  has  $y$ -intercept equal to  $(0, 1)$ .

b) Find the vertical and horizontal asymptotes, if any, of the curve

$$y = \frac{(2x + 1)^2}{x^2 - 3x + 2}$$

3. Compute the following limits

a)

$$\lim_{x \rightarrow 1} \frac{2 - \sqrt{3x + 1}}{1 - x}$$

b)

$$\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\theta \sin \theta}$$

4. a) Using the definition of the derivative as a limit (only method allowed), compute the derivative of the function

$$f(x) = 2x^2 + \frac{1}{x+1}$$

when  $x \neq -1$ .

b) Find all values of  $x$  where the curve  $y = \cos(x^3)$  has a horizontal tangent line.

5. a) Find the second derivative of  $y = \tan x$ .

b) Let  $y = x \cos x$ . If  $y'(x_0) = 0$ , first show that  $y(x_0) \neq 0$ , and then find  $\tan(x_0)$  in terms of  $x_0$ .

6. a) Find  $dy/dx$  if

$$y = \sqrt{2 + \sin(4x)}$$

b) A rocket is fired, at time  $t = 0$ , from a tower straight up into the air. Its height above ground level at time  $t$  is given by

$$s(t) = 96 + 16t - 16t^2$$

How tall is the tower and what is the initial velocity of the rocket? When does the rocket reach ground level and what is its speed at that time?

7. a) Find the slope of the tangent line to the curve

$$y = 4 \sin^3 2x$$

at  $x = \pi/6$ .

b) Use the Sandwich Theorem to evaluate the limit

$$\lim_{x \rightarrow 0} x^2 \sin \frac{1}{x}$$