CALCULUS 221

6th WEEK EXAM

I. M. Isaacs
Friday, October 10, 2008
5:30 – 7:00 P.M.

Do all problems — 100 points.
Use backs of pages for scrap, or if you need more space.

NAME: _____

TA: _____

Do not write below here.

Prob. 1: _____ out of   20.

Prob. 2: _____ out of   20.

Prob. 3: _____ out of   15.

Prob. 4: _____ out of   16.

Prob. 5: _____ out of   7.

Prob. 6: _____ out of   10.

Prob. 7: _____ out of   12.

Total: _____ out of   100.

1. [20 POINTS] Compute dy/dx for each of the following.

(a) $y = \dfrac{\sin(3x)}{x^2}$.

(b) $y = \dfrac{1}{\sqrt[3]{1 - x^2}}$.

(c) $y = \sec^3(4x^2)$.

(d) $y = x(1 + x)^3(1 - x)^5$.

2. [20 POINTS] For each of the following, compute dy/dx at the indicated point.

(a) $y = v^3 + v^2$; $x = 5 - v^2$, at the point where $v = 1$.

(b) $\dfrac{xy}{x+y-1} = y - 1$, at the point $(4, 3)$.

(c) $y = S(\pi - x)$ at the point $x = 2\pi/3$, given that $S'(x) = \sec x$.

(d) $y = g(x)$ at the point where $x = 1$, given that $h(x) = (g(x))^2$ and $h'(1) = 12$, and that $g(1) = 2$.

3. [15 POINTS] The functions $f(x)$ and $g(x)$ are differentiable and the following table gives some of the values of these functions and their derivatives. Use these data to compute the following items. If there is not enough information given to determine some item, write "UNKNOWN".

x	-1	2	3	5	7
$f(x)$	2	3	5	-1	-3
$f'(x)$	1	4	7	3	-2
$g(x)$	3	2	1	2	3
$g'(x)$	5	-3	-2	7	5

(a) $F(5)$, where $F(x) = g(f(x))$.

(b) $F'(5)$, where $F(x) = g(f(x))$.

(c) $G'(1)$, where $G(x) = f(2x)g(3x)$.

4. [16 POINTS] Compute each of these limits. If the limit does not exist, write "DNE".

(a) $\displaystyle\lim_{x \to 1^-} \frac{x^3}{x^2 - 1}$

(b) $\displaystyle\lim_{y \to 2} \frac{y^2 + y - 6}{2 - y}$

(c) $\displaystyle\lim_{x \to \infty} \frac{\sqrt[3]{8x^6 + x^5}}{x^2 + x + 1}$

(d) $\displaystyle\lim_{t \to 0} \frac{\sec^2(\frac{\pi}{4} + t) - 2}{t}$

5. [7 POINTS] Let
$$f(x) = \begin{cases} 1/x & \text{if } 0 < x < 1 \\ 2 & \text{if } x = 1 \\ x & \text{if } x > 1. \end{cases}$$

Find a number a such that for every choice of $\epsilon > 0$, there exists $\delta > 0$ such that $|f(x) - a| < \epsilon$ whenever $0 < |x - 1| < \delta$. Also, if $\epsilon = 1/3$, find the largest possible δ with this property.

Sketch a graph that shows what is going on here.

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6. [10 POINTS] A particle moves along the x-axis in such a way that at time t, its position is $x = 12t - t^3$.

(a) Find the velocity at time $t = 0$, the velocity at time $t = 3$ and the average velocity during the interval $0 \le t \le 3$

(b) Find the time $t > 0$ when the velocity is 0, and find the acceleration at that time.

7. [12 POINTS] The point $(1,1)$ lies on the curve described by the equation $x^2 + xy + y^2 = 3$.

(a) Find the equation of the line tangent to the curve at $(1,1)$

(b) Find d^2y/dx^2 at the point $(1,1)$.

(c) Find all points on the curve other than $(1,1)$ where the tangent is parallel to the tangent at $(1,1)$.

THE END