

Show all work. **Circle your answer.**

No books, no notes, no electronic devices at all.

Solutions will be posted shortly after the exam: www.math.wisc.edu/~miller/m217

Name_____

Hand in to your TA – Circle your DIScussion section (column one):

DIS 321	7:45 TR	901 Van Vleck	Forray, Susan Jane
DIS 322	8:50 TR	6224 Soc Sci	Bisgard, James William
DIS 324	9:55 TR	123 Ingraham	Bisgard, James William
DIS 326	11:00 TR	B235 Van Vleck	Leete, Jennifer A.
DIS 327	11:00 TR	113 Ingraham	Lu, Tianshi
DIS 328	12:05p TR	B235 Van Vleck	Leete, Jennifer A.
DIS 329	12:05p TR	6228 Soc Sci	Lu, Tianshi
DIS 330	1:20p TR	B321 Van Vleck	Forray, Susan Jane
DIS 332	2:25p TR	B135 Van Vleck	Hur, Youngmi
DIS 334	3:30p TR	B329 Van Vleck	Hur, Youngmi

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total	60	

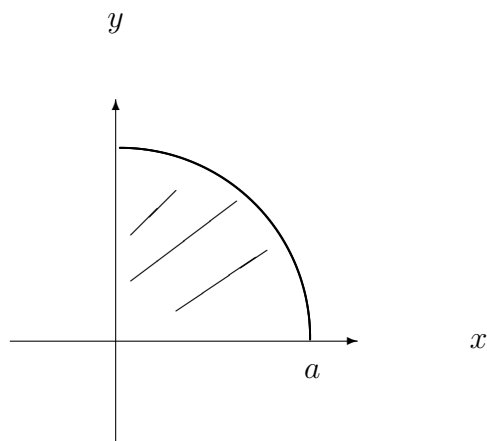
1. (10 pts) Sketch the ellipse with equation:

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

Label all four points where the ellipse intersects the axis.
Find, draw, and label the focal points.

2. (10 pts) Find the center of mass of a thin homogeneous plate covering the first quadrant of the disk bounded by the circle $x^2 + y^2 = a^2$.

Circle your answer.



3. (10 pts) The amount of \$4000 is invested at the interest rate of 6% compounded monthly. How much will you have after 3 years and 4 months?

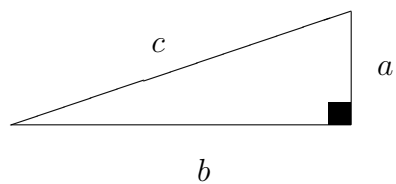
Circle your answer.

4. (10 pts) A roasted Turkey is taken from an oven when its temperature has reached 170 degrees and placed on a table in a room where the temperature is 70 degrees. If the temperature of the Turkey is 150 degrees after 10 minutes, when will its temperature be 140 degrees?

Circle your answer.

5. (10 pts) Prove Pythagorus's Theorem: In a right triangle with hypotenus c and sides a and b , we have that

$$a^2 + b^2 = c^2$$



6. (10 pts) (A) (Do either problem 6A or problem 6B. **Do not do both.**)

Prove the following part of the fundamental theorem of calculus:

Suppose $f(x)$ is continuous on the interval $[a, b]$, $F(x)$ is continuous on $[a, b]$ and differentiable on (a, b) with the $F'(x) = f(x)$ for all x in (a, b) . Let n be a large integer.

Define $\Delta x = \frac{b-a}{n}$.

Define $x_k = a + k\Delta x$ for $k = 0, 1, \dots, n$.

Show there exists c_k with $x_{k-1} \leq c_k \leq x_k$ for $k = 1, 2, \dots, n$ and such that

$$F(b) - F(a) = \sum_{k=1}^n f(c_k) \Delta x$$

6. (10 pts) (B) (Do either problem 6A or problem 6B. **Do not do both.**)

Prove the following version of the fundamental theorem of calculus:

Suppose $f(x)$ is a continuous function on the interval $[a, b]$ and $f(x) \geq 0$ for all x in $[a, b]$. For any $x \in [a, b]$ let $A(x)$ be the area under f and above the interval $[a, x]$ on the x -axis.

Prove

$$\frac{dA}{dx} = f(x)$$

Answers

1. The focal points are at $(\pm 4, 0)$. The ellipse intersects the axes at $(\pm 5, 0)$ and $(0, \pm 3)$.

2. $(\frac{4}{3\pi}a, \frac{4}{3\pi}a)$

3. $4000(1 + \frac{.06}{12})^{40} = 4000(1.005)^{40} = 4883.1769$ dollars

4. $10^{\frac{\ln(.7)}{\ln(.8)}} = 15.984103$ minutes

5. See Exam 3 problem 6.

6. See Exam 1 problem 6 for 6A. For 6B see pages 196-198 of your calculus book:
We must prove that

$$\lim_{\Delta x \rightarrow 0} \frac{A(x + \Delta x) - A(x)}{\Delta x} = f(x)$$

We consider the limit for $\Delta x \rightarrow 0^+$ the proof for $\Delta x \rightarrow 0^-$ is similar. Let $\Delta x > 0$ be small. Then $A(x + \Delta x) - A(x)$ is the area below f and above the line segment $[x, x + \Delta x]$ on the x -axis. Let m be the minimum value of f on $[x, x + \Delta x]$ and M the maximum value. Then it is clear that

$$m\Delta x \leq A(x + \Delta x) - A(x) \leq M\Delta x$$

Hence

$$m \leq \frac{A(x + \Delta x) - A(x)}{\Delta x} \leq M$$

By the extreme value theorem and intermediate value theorem for continuous functions there exists x^* with $x \leq x^* \leq x + \Delta x$ such that

$$f(x^*) = \frac{A(x + \Delta x) - A(x)}{\Delta x}$$

Since f is continuous

$$\lim_{\Delta x \rightarrow 0^+} f(x^*) = f(x)$$

Hence

$$\lim_{\Delta x \rightarrow 0^+} \frac{A(x + \Delta x) - A(x)}{\Delta x} = \lim_{\Delta x \rightarrow 0^+} f(x^*) = f(x)$$