

Show all work.

No books, no notes, no calculator, no cell phones, no pagers, no electronic devices at all.

Solutions will be posted shortly after the exam: www.math.wisc.edu/~miller/m217

Name_____

Hand in to your TA – Circle your DIScussion section (column one):

DIS 321	7:45 TR	B309 Van Vleck	Forray, Susan Jane
DIS 322	8:50 TR	6224 Soc Sci	Bisgard, James William
DIS 324	9:55 TR	123 Ingraham	Bisgard, James William
DIS 326	11:00 TR	123 Ingraham	Leete, Jennifer A.
DIS 327	11:00 TR	113 Ingraham	Lu, Tianshi
DIS 328	12:05p TR	4322 Soc Sci	Leete, Jennifer A.
DIS 329	12:05p TR	6228 Soc Sci	Lu, Tianshi
DIS 330	1:20p TR	6310 Soc Sci	Forray, Susan Jane
DIS 332	2:25p TR	B135 Van Vleck	Hur, Youngmi
DIS 334	3:30p TR	B329 Van Vleck	Hur, Youngmi

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total	60	

1. (10 pts) A square sheet of tin 12 inches on a side is used to make an open-top box by cutting a small square of tin from each corner and bending up the sides. How large a square should be cut from each corner in order that the box shall have as large a volume as possible?

Let x be the length of the side of the small square. Give your answer in the form $x = \dots$

2. (10 pts) Solve the following differential equation:

$$\frac{dy}{dx} = 3x^2y^2$$

Give your answer in the form $y = \dots$

3. (10 pts) Evaluate the following indefinite integral:

$$\int \sin^3(3x) \cos(3x) dx$$

4. (10 pts) Establish the formula

$$\sum_{k=1}^n (2k - 1) = 1 + 3 + 5 + \cdots + (2n - 1) = n^2$$

for every positive integer n , by showing

(a) the formula is correct for $n = 1$, and

(b) if true for n , the formula is also true for $n + 1$.

5. (10 pts) Find the area A between the curve $y = 9 - x^2$ and the x -axis. Give your answer in the form $A = \dots$

6. (10 pts) Prove the following part of the fundamental theorem of calculus:

Suppose $f(x)$ is continuous on the interval $[a, b]$, $F(x)$ is continuous on $[a, b]$ and differentiable on (a, b) with the $F'(x) = f(x)$ for all x in (a, b) . Let n be a large integer.

(a) Define $\Delta x =$

(b) Define $x_k =$

(c) Show there exists c_k with $x_{k-1} \leq c_k \leq x_k$ for $k = 1, 2, \dots, n$ and such that

$$F(b) - F(a) = \sum_{k=1}^n f(c_k) \Delta x$$

Answers

1. The volume of the box is $V = x(12 - 2x)^2$ and is a maximum on $[0, 12]$ when $x = 2$.

2. $y = \frac{-1}{x^3+C}$ Remark: there is also the constant solution $y \equiv 0$.

3. $\int \sin^3(3x) \cos(3x) dx = \frac{\sin^4(3x)}{12} + C$

4. (a) $\sum_{k=1}^1 (2k - 1) = 2 \cdot 1 - 1 = 1 = 1^2$

(b) $\sum_{k=1}^{n+1} (2k - 1) = \sum_{k=1}^n (2k - 1) + (2(n+1) - 1) = n^2 + (2(n+1) - 1) = n^2 + 2n + 1 = (n+1)^2$

5. $A = 36$

6. (a) $\Delta x = \frac{b-a}{n}$

(b) $x_k = a + k\Delta x$

(c) Using the Mean Value Theorem we have that for every k there exists c_k with $x_{k-1} < c_k < x_k$ and

$$F'(c_k) = \frac{F(x_k) - F(x_{k-1})}{x_k - x_{k-1}}$$

Noting that $F' = f$ and $\Delta x = x_k - x_{k-1}$ we have that

$$f(c_k) = F'(c_k) = \frac{F(x_k) - F(x_{k-1})}{x_k - x_{k-1}} = \frac{F(x_k) - F(x_{k-1})}{\Delta x}$$

and hence

$$f(c_k)\Delta x = F(x_k) - F(x_{k-1})$$

Hence,

$$\sum_{k=1}^n f(c_k)\Delta x = \sum_{k=1}^n F(x_k) - F(x_{k-1})$$

$$= (F(x_1) - F(x_0)) + (F(x_2) - F(x_1)) + (F(x_3) - F(x_2)) + \cdots + (F(x_n) - F(x_{n-1}))$$

This is a telescoping sum in which everything cancels except $-F(x_0)$ and $F(x_n)$

$$= F(x_n) - F(x_0) = F(b) - F(a)$$

Noting that $x_0 = a$ and $x_n = b$ finishes the proof.

$$\sum_{k=1}^n f(c_k)\Delta x = F(b) - F(a)$$