Show all work. Circle your answer.

You may use your cheat sheet, one 8.5×11 inch paper with anything you want written on either side.

Otherwise, no books, no calculator, no cell phones, no pagers, no electronic devices at all.

Solutions will be posted shortly after the exam: www.math.wisc.edu/~miller/m213

Name______________________________

Circle your DIScussion section:

TA: Youngsuk Lee

DIS 301	8:50 T	6322 SOC SCI
DIS 302	8:50 R	215 INGRAHAM
DIS 303	9:55 T	225 INGRAHAM
DIS 304	9:55 R	495 VAN HISE

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

1. (10 pts) Solve the differential equation:

$$\frac{dy}{dx} = \frac{\ln(x+1)\sqrt{y}}{e^{\sqrt{y}}}$$

Find an equation relating x and y, you need not explicitely solve for y.

Circle your answer.

2. (10 pts) Find

$$\int \frac{\ln(x)}{x} dx$$

Circle your answer.

3. (10 pts) Find the critical points of the function and classify each as either saddle points or relative (or local) maximums or minimums.

$$f(x, y) = x^2 + 4xy + y^2 + 6x + 2$$

Circle your answer.

4. (10 pts) The graph of the function $y = x^3$ for x such that $0 \leq x \leq 1$ is rotated around the x-axis, i.e. $y = 0$. Find the volume of the solid of rotation.

Circle your answer.

5. (10 pts) A rectangular box with a square bottom and no top is to be built so as to have volume 48 cubic inches. The cost of the material for making the square bottom is 3 cents per square inch. The cost of the material for making the four sides is 2 cents per square inch. Find the dimensions of the box, ie. bottom $x \times x$ and height y which minimize the cost.

Circle your answer.

6. (10 pts) Use Euler's method to find an approximation solution to

$$\frac{dy}{dx} = 1 + y \text{ and } y = 0 \text{ when } x = 1$$

Use a step size of $h = \triangle x = \frac{1}{10} = .1$ to find the approximate value of y when $x = 1.2$.

Circle your answer.

7. (10 pts) Evaluate the double integral below. The function below is impossible to integrate symbolically as it is.

$$\int_0^1 \int_y^1 e^{x^2} \, dx \, dy$$

(a) Draw the region A over which the integration takes place. Shade the region A.
(b) Describe A in two different ways, i.e. $A = \{(x, y) : ??? \}$
(c) Interchange the limits of integration.
(d) Integrate.

Circle your answer.

8. (10 pts) Tamara wants to buy a boat that she estimates will cost \$2,000 when she buys it. How much money must she deposit at the beginning of each month in order to have enough after one year to pay for her boat? Assume $\frac{1}{2}\%$ interest per month compounded monthly (6% annual). The first payment is made on Jan 1, 2002, the second on Feb 1, 2002, and so forth until the last payment is made on Jan 1 2003, and the boat is bought on that day for \$2,000. What is her monthly payment p?

Circle your answer.

9. (10 pts)

(a) Find the Taylor series for the function:

$$f(x) = \frac{4}{4 - x^2}$$

(b) Determine the radius of convergence r, converges for all x with $-r < x < r$.

Circle your answer.

10. (10 pts)

(a) Find the Taylor polynomial of degree two near zero for the function:

$$f(x) = \sqrt{16 + x}$$

(b) Use this polynomial to approximate

$$\sqrt{17}$$

Circle your answer.

Answers

1.
$$2e^{\sqrt{y}} = (x+1)\ln(x+1) - x + C$$

2.
$$\frac{(\ln(x))^2}{2} + C$$

3. Saddle at $(1, -2)$.

4. $\frac{\pi}{7}$

5. $4 \times 4 \times 3$

6. .21

7. (d) $\frac{1}{2}(e-1)$

8. There are 13 payments. For $i = .005$

$$p + p(1+i) + p(1+i)^2 + \cdots + p(1+i)^{12} = 2000$$

so $p(\frac{1-x^{13}}{1-x}) = 2000$ for $x = (1+i)$ and hence $p = 2000(\frac{1-x}{1-x^{13}})$

9.
$$\frac{4}{4-x^2} = 1 + \frac{1}{2}x^2 + \frac{1}{4}x^4 + \cdots = \sum \frac{1}{2^n}x^{2n}$$
It converges for all x such that $-2 < x < 2$, radius is $r = 2$.

10.
(a) $p(x) = 4 + \frac{1}{8}x - \frac{1}{512}x^2$
(b) $4\ \frac{63}{512}$