

Calculators can be used, but will not be needed. (Graders prefer no decimals in your answer.)

You may bring one "Cheat Sheet", a single 8 1/2 x 11 sheet of paper with anything you want written on both sides.

Show all your work. Write clearly and carefully. Little partial credit will be given for the correct answer with incorrect or no work. Cross out any work or answers which you do not want graded. Circle your answer. Hand in your exam to your TA.

Name_____

Circle your TA section:

211 002 321	Lang, Mike	7:45	TR	B333	Van Vleck
211 002 322	Lang, Mike	8:50	TR	202	Bascom
211 002 324	Rhee, Kyoungwon	9:55	TR	301	Educ Sci
211 002 326	Rhee, Kyoungwon	11:00	TR	3331	Sterling
211 002 327	Hartwig, Brian	12:05	TR	346	Birge
211 002 328	Lee, Byungyoon	12:05	TR	341	Van Vleck
211 002 329	Lee, Byungyoon	1:20	TR	341	Van Vleck
211 002 331	Hartwig, Brian	2:25	TR	114	Ingraham
211 002 332	Kethireddypally	2:25	TR	223	Ingraham
211 002 334	Kethireddypally	3:30	TR	B325	Van Vleck

Problem	Points	Score
1	8	
2	8	
3	8	
4	8	
5	8	
6	8	
7	8	
8	8	
9	8	
10	8	
Total	80	

1. (8 pts) A person initially places \$1000 in a savings account that pays 10% per year compounded continuously. The person makes additional deposits of \$100 per year continuously through each year.

Find a differential equation for $P(t)$, the amount on deposit after t years. Solve it.

2. (8 pts) Duck soup production in the United States increased from 12 million bowls in 1991 to 18 million bowls in 1995. Assuming production of soup to follow a linear equation, find the production level predicted by these data for the year 2000.

3. (8 pts)

The cost function $C(x)$ for producing x barrels of kippered herring is defined by

$$C(x) = \begin{cases} 4x^2 + 8 & 0 < x < 2 \\ 2x^2 + 8x & 2 \leq x < 3 \\ x^2 + 14x & 3 \leq x < 5 \end{cases}$$

At which points is $C(x)$ continuous? differentiable? What is the marginal cost function, $MC(x)$?

4. (8 pts) Mrs. Claypool currently sells 100 subscriptions annually for the New York Opera at a rate of \$40 per year. Otis P. Driftwood, the entrepreneur, predicts that for each \$1 reduction in subscription price an additional 5 subscriptions can be sold. Under this assumption what subscription rate maximizes revenue for the Opera?

5. (8 pts) Your greatuncle Cuthbert J. Twillie has died and left you a spendthrift trust. Uncle Cuthbert believed that too much money would kill your incentive to work hard, so he arranged that the trust would pay you nothing until your 40th birthday (assume you are 20) and after which would pay you \$500 each year for the next ten years. The money is to be paid out as a weekly allowance by the NGSEB Trust Company but you may assume that the revenue stream is continuous. Assuming prevailing interest rates are 10% compounded continuously, what amount should NGSEB be willing to pay you today in return for these future funds?

6. (8 pts) Dr.Hugo Z.Hackenbush, veteran veterinarian, experiences weekly costs of

$$C(x) = 4 + 3x + e^{\frac{x}{5}}$$

in producing x horse tranquilizer pills. If the pills can be sold for \$33 each, find the weekly production level of pills that maximizes profits.

7. (8 pts) The population of the country of Freedonia is presently 6 thousand people. This number is expected to double to 12 thousand in the next 30 years. Assume that the population of Freedonia is given by an exponential equation. What will the average population of Freedonia be during this 30 year period?

8. (8 pts) Find all critical points and classify them relative minimum, relative maximum, or saddle for the function

$$f(x, y) = x^2 - y^2 + 6x + 4y + 5$$

9. (8 pts) Find the partial derivatives z_x and z_y for

$$z = \frac{x}{y^2} - \frac{y}{x^2}.$$

10. (8 pts) The prices p and q of two items are related to the number of items, x and y , that will be sold during a fixed period of time by the demand equations:

$$p = 20 - x \qquad q = 46 - 4y$$

The total cost of producing x items of the first kind and y items of the second kind during this period of time is:

$$C(x, y) = 100 + 7x + 2y + xy$$

Find number of items, x and y , for which profit will be a maximum.

Answers

1. $P' = .10P + 100$ and $P(0) = 1000$ so $P(t) = 2000e^{.10t} - 1000$
2. $25\frac{1}{2}$ million bowls.
Duck Soup, Marx Bros 1933.
3. $C(x)$ is continuous in $(0, 3)$ and $(3, 5)$ (not at the point $x = 3$). It is differentiable at the same set of points.

$$MC(x) = \begin{cases} 8x & 0 < x < 2 \\ 4x + 8 & 2 \leq x < 3 \\ 2x + 14 & 3 < x < 5 \\ \text{undefined} & x = 3 \end{cases}$$

Monkey Business, Marx Bros, 1931.

4. \$30

A Night at the Opera, Marx Bros, 1935.

5. $5000(e^{-2} - e^{-3})$

NGSEB, Never Give a Sucker an Even Break, W.C.Fields 1941.

6. $5 \ln(150)$

A day at the races, Marx Bros, 1937.

7. $\frac{6}{\ln(2)}$

Duck Soup, Marx Bros 1933.

8. Saddle point at $(-3, 2)$

9. $z_x = \frac{1}{y^2} + 2\frac{y}{x^3}$ and $z_y = -2\frac{x}{y^3} - \frac{1}{x^2}$

10. $x = 4$ and $y = 5$.