

No calculators. No notes. No books.

Show all your work. Write clearly and carefully.

Little partial credit will be given for the correct answer with incorrect or no work.

Cross out any work or answers which you do not want graded.

Circle your answer.

Hand in your exam to your TA.

Name_____

Circle your TA section:

211 002 321	Lang, Mike	7:45	TR	B333	Van Vleck
211 002 322	Lang, Mike	8:50	TR	202	Bascom
211 002 324	Rhee, Kyoungwon	9:55	TR	301	Educ Sci
211 002 326	Rhee, Kyoungwon	11:00	TR	3331	Sterling
211 002 327	Hartwig, Brian	12:05	TR	346	Birge
211 002 328	Lee, Byungyoon	12:05	TR	341	Van Vleck
211 002 329	Lee, Byungyoon	1:20	TR	341	Van Vleck
211 002 331	Hartwig, Brian	2:25	TR	114	Ingraham
211 002 332	Kethireddypally	2:25	TR	223	Ingraham
211 002 334	Kethireddypally	3:30	TR	B325	Van Vleck

Problem	Points	Score
1	4	
2	4	
3	4	
4	4	
5	4	
6	4	
7	4	
8	4	
9	4	
10	4	
Total	40	

1. Find $\frac{dy}{dx}$ by logarithmic differentiation:

$$y = \frac{x(x-2)(x+3)}{(x-4)(x+5)}$$

2. A manufacturer experiences weekly costs of

$$C(x) = 4000 + 500x + e^{5x}$$

in producing x items. If the items can be sold for \$1000 each, find the weekly production level x that maximizes profits.

3. When a foreign substance is introduced into the body, the body's defense mechanisms move to break down the substance and excrete it. The rate of excretion is usually proportional to the concentration in the body, and the half-life of the resulting exponential decay is referred to as the *biological half-life* of the substance.

If, after 8 hours, 10% of a massive dosage of a substance has been excreted by the body, what is the biological half-life of the substance?

4. Find the area bounded by the curves $y = x^3$ and $y = x^2$.

5. Find the consumer's surplus for the demand function,

$$p = D(x) = \frac{8}{x+1}$$

when the **price** is \$2.

6. An investor wishes to find an annuity capable of producing a continuous revenue stream of \$50000 per year for 10 years, beginning 15 years from now. What is the present value of this revenue stream, assuming an interest rate of 6%?

7. The population of the earth is presently 6 billion people. This number is expected to grow exponentially to 12 billion in the next 30 years. What will be the average population of the earth during this 30 year period?

8. Find the maximum and minimum values of

$$f(x) = x + \cos(x)$$

in the interval $[0, \pi]$.

9. Find

$$\int_0^{\sqrt{\pi/2}} x \sin^4(x^2) \cos(x^2) \, dx$$

10. Prove the Fundamental Theorem of Calculus:

Let f be continuous on $[a, b]$ and let F be any antiderivative for f , i.e., $F'(x) = f(x)$ for all x . Then

$$\int_a^b f(x)dx = F(b) - F(a)$$

Answers

1. $\frac{x(x-2)(x+3)}{(x-4)(x+5)} \left(\frac{1}{x} + \frac{1}{x-2} + \frac{1}{x+3} - \frac{1}{x-4} - \frac{1}{x+5} \right)$

2. $x = \frac{1}{5} \ln(100)$

3. $\frac{\ln(\frac{1}{2})}{\frac{1}{8} \ln(\frac{9}{10})}$

4. $\frac{1}{12}$

5. $8 \ln(4) - 6$

6. $\frac{50000}{(-.06)} (e^{-(.06)25} - e^{-(.06)15})$

7. $\frac{6}{\ln(2)}$ billion

8. minimum 1 at $x = 0$, maximum $\pi - 1$ at $x = \pi$.

9. $\frac{1}{10}$

10. Textbook page 408