No books, no notes, no electronic devices of any kind. Show all your work. Simplify your answer and circle it.

Name_____

Circle your TA's name and the time your section starts.

Clement, Nathan	M	11:00	1:20
Jefferis, Leland	M	1:20	2:25
Makuluni, Edson	T	8:50	9:55
Nan, Ting-Ting	T	11:00	12:05
Wang, Kejia	M	8:50	9:55
Yao, Chengjian	T	1:20	2:25
You, Qian	T	11:00	12:05
Zhao, Yongqiang	M	8:50	11:00

Hand in to your TA.

Detach pages 14-16 from your exam and take them home with you.

Hand in only pages 0-13.

Page	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
13	10	
14	5	
15	5	
16	5	
17	5	
Total	150	

1. (10 pts)

$$f(x) = x^2$$

Find the derivative of f using the definition of derivative. Begin by stating the definition of $f'(x)$. Do **not** use differentiation rules. This problem is from Section 2.1. In all other problems you may use the differentiation rules to compute derivatives.

2. (10 pts) Suppose y is implicitely defined as a function of x by the equation:

$$e^{xy} + x + y = 2$$

Find $\frac{dy}{dx}$ when $x = 0$ and $y = 1$. Circle you answer.

3. (10 pts) Note that this is the derivative of f:

$$f'(x) = x^2(x-1)^3(e^x+1)$$

Find the intervals where f is increasing or decreasing. Circle your answer.

4. (10 pts) Suppose q is the demand and p is the price of a certain commodity. The elasticity of demand, E, is the percentage change of q divided by the percentage change of p taken to the limit:

$$E = \lim_{\triangle p \to 0} \frac{\frac{\triangle q}{q}}{\frac{\triangle p}{p}}$$

If the demand is related to the price by $q = 300 - p^2$, find the price p for which $E = -1$, i.e., of unit elasticity. Circle your answer.

5. (10 pts)

$$D(q) = 450 - q^2 \qquad S(q) = 150 + 2q^2$$

are the demand and supply functions for a particular commodity. Specifically q units will be demanded (sold) at a price $p = D(q)$ dollars per unit, while q units will be supplied by producers when the price is $p = S(q)$ dollars per unit.

(a) Find the equilibrium point (q_e, p_e) where supply equals demand.

(b) Find the producer's surplus at equilibrium.

Circle your answers.

6. (10 pts) Find the value of the definite integral:

$$\int_0^1 \frac{x}{e^{2x}}\, dx$$

7. (10 pts) Find the particular solution of the differential equation

$$\frac{dy}{dx} = \frac{2x}{y^2}$$

satisfying $y = 3$ when $x = 2$. Circle your answer.

8. (10 pts) Find the second partials (including the mixed partials) of

$$z = e^{xy^2}$$

Circle your answers.

9. (10 pts) Find the critical points of

$$f(x.y) = x^3 - 6xy + y^3$$

and classify each as a relative minimum, relative maximum, or saddle point. Circle your answer.

10. (10 pts) Given the points

$$(1, 2) \quad (2, 4) \quad (4, 4)$$

find the least-squares line. Circle your answer.

11. (10 pts) Find the minimum and maximum values of

$$f(x, y, z) = x + 2y + 3z$$

subject to the constraint $x^2 + y^2 + z^2 = 14$. Circle your answers.

12. (10 pts) Find the volume of the solid under the surface:

$$f(x, y) = 2x + 2y$$

and over the region R where R is bounded by $y = x$, $y = 2 - x$, and $y = 0$. Circle your answer.

Detach pages 14-16 from your exam and take them home with you.
Hand in only pages 0-13.
Record your answers to problems 13-17 here.

Circle your answers below.

13.

(a) 0 1 2 ∞ $-\infty$ doesn't-exist none-of-these

(b) 0 1 2 ∞ $-\infty$ doesn't-exist none-of-these

(c) 0 1 2 ∞ $-\infty$ doesn't-exist none-of-these

(d) 0 1 2 ∞ $-\infty$ doesn't-exist none-of-these

(e) 0 1 2 ∞ $-\infty$ doesn't-exist none-of-these

(f) 0 1 2 ∞ $-\infty$ doesn't-exist none-of-these

(g) 0 1 2 ∞ $-\infty$ doesn't-exist none-of-these

(h) 0 1 2 ∞ $-\infty$ doesn't-exist none-of-these

(i) 0 1 2 ∞ $-\infty$ doesn't-exist none-of-these

(j) 0 1 2 ∞ $-\infty$ doesn't-exist none-of-these

14. a b c d e

15. a b c d e

16. a b c d e

17. a b c d e

Put your answers on page 13.

13. (10 pts)

(a) $\lim_{x\to\infty} \frac{1}{x}$

(b) $\lim_{x\to-\infty} \frac{1}{x}$

(c) $\lim_{x\to0^+} \frac{1}{x}$

(d) $\lim_{x\to0^-} \frac{1}{x}$

(e) $\lim_{x\to0} \frac{1}{x}$

(f) $\lim_{x\to0} \frac{1}{x^2}$

(g) $\lim_{x\to\infty} e^x$

(h) $\lim_{x\to-\infty} e^x$

(i) $\lim_{x\to0^+} \ln(x)$

(j) $\lim_{x\to\infty} \ln(x)$

14. (5 pts) The improper integral $\int_2^\infty \frac{1}{x^2}$

a. is ∞.
b. is 1.
c. diverges but not to ∞.
d. is $-\frac{1}{2}$.
e. none of above

Put your answers on page 13.

15. (5 pts) The value of the trapazoid rule to estimate the integral $\int_1^2 \frac{1}{x}\,dx$ with $n = 4$ is

a. undefined
b. ∞
c. $\frac{320987}{22}$
d. $\ln(2)$
e. none of above

16. (5 pts) For what value of A (if any) is the following function f continuous for every x?

$$f(x) = \begin{cases} \ln(x) & \text{if } x > 1 \\ x^2 + Ax - 2 & \text{if } x \leq 1 \end{cases}$$

a. $A = 1$
b. no A
c. all $A \leq 1$
d. $A = e$
e. none of above.

17. (5 pts) Describe the domain of the function:

$$f(x, y) = \sqrt{y - x^2}$$

a. All ordered pairs (x, y) of real numbers.
b. All real numbers y which are nonnegative.
c. All ordered pairs (x, y) of real numbers such that $y \geq x^2$.
d. All ordered pairs (x, y) of real numbers such that $y = x^2$.
e. None of above.

Use this sheet for scratch paper.

Answers

1. Define
$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
then
$$f(x+h) - f(x) = (x+h)^2 - x^2 = x^2 + 2hx + h^2 - x^2 = 2hx + h^2$$
so
$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{2xh + h^2}{h} = \lim_{h\to 0} 2x + h = 2x$$

2. -2

3. decreasing for $x < 1$ and increasing for $x > 1$.

4. $p = 10$

5. $(10, 350)$ and $PS = \frac{4000}{3}$

6. $\frac{1}{4} - \frac{3}{4}e^{-2}$

7. $y = (3x^2 + 15)^{\frac{1}{3}}$

8.
$$\frac{\partial^2 z}{\partial x \partial y} = 2y(y^2x + 1)e^{xy^2}$$

9. rel min at $(2, 2)$, saddle at the origin.

10. $y = \frac{4}{7}x + 2$

11. max 14, min -14

12. $\frac{8}{3}$

13. 0 0 ∞ $-\infty$ doesn't-exist ∞ ∞ 0 $-\infty$ ∞ 14e 15e 16a 17c