No books, no notes, no electronic devices of any kind. Show all your work. Simplify your answer and circle it.

Name_____

Circle your TA's name and the time your section starts.

Clement, Nathan	M	11:00	1:20
Jefferis, Leland	M	1:20	2:25
Makuluni, Edson	T	8:50	9:55
Nan, Ting-Ting	T	11:00	12:05
Wang, Kejia	M	8:50	9:55
Yao, Chengjian	T	1:20	2:25
You, Qian	T	11:00	12:05
Zhao, Yongqiang	M	8:50	11:00

Hand in to your TA.

Page	Points	Score
1	7	
2	7	
3	7	
4	7	
5	7	
6	7	
7	7	
8	7	
9	7	
10	8	
11	7	
12	7	
13	7	
14	8	
Total	100	

1. (7 pts) The region under the curve

$$y = \frac{1}{\sqrt{x}}$$

for x such that $1 \leq x \leq 2$ is rotated about the x axis. What is the volume of the solid of revolution?

Circle your answer.

2. (7 pts) Solve the following equation for x:

$$2\ln(x) = \ln(12) - \ln(3)$$

Simplify your answer. Circle your answer.

3. (7 pts) Determine the critical numbers of the function:

$$g(t) = \frac{t^2}{t^2 + t + 2}$$

Using the first derivative only, determine whether each critical point is a relative minimum, relative maximum, or neither. Circle your answer.

4. (7 pts) Use the second derivative test to find the relative maxima and minima of f:

$$f(x) = x + 1 + \frac{4}{x}$$

Circle your answer.

5. (7 pts) Suppose q is the demand and p is the price of a certain commodity. The elasticity of demand, E, is the percentage change of q divided by the percentage change of p taken to the limit:

$$E = \lim_{\triangle p \to 0} \frac{\frac{\triangle q}{q}}{\frac{\triangle p}{p}}$$

If the demand is related to the price by $q = 100 - p^2$ for $0 \leq p \leq 10$, find the price p for which $E = -2$, i.e., a small percentage raise in price leads to a drop in demand by twice that percentage. Circle your answer.

6. (7 pts) It is estimated that the cost of constructing an office building that is n floors high is

$$C(n) = 2n^2 + 500n + 200$$

thousand dollars. How many floors should the building have in order to minimize the average cost per floor? Circle your answer.

7. (7 pts) How much money should be invested now at 5% nominal annual interest to obtain \$10000 in 10 years if interest is compounded continuously? Circle your answer.

8. (7 pts) Find $f'(x)$ given that

$$f(x) = \frac{x}{2^x}$$

Circle your answer.

9. (7 pts) Graph the function

$$f(x) = xe^x$$

Determine where it is increasing or decreasing, concave up or down. Show as many key features as possible: high and low points, points of inflection, vertical and horizontal asymptotes, intercepts, cusps, vertical and horizontal tangents.

10. (8 pts) Solve

$$\frac{dy}{dx} = \frac{1}{x} - \frac{2}{x^2} \qquad \text{where } y = 5 \text{ when } x = 1.$$

Circle your answer.

11. (7 pts) Find the integral:

$$\int \frac{4x}{2x+1}\,dx$$

Circle your answer.

12. (7 pts) Find

$$\int_1^2 2f(x) + 5g(x) \ dx$$

given that

$$\int_0^2 f(x)\ dx \ = 5 \qquad \int_0^2 g(x)\ dx \ = 2 \qquad \int_0^1 f(x)\ dx \ = 1 \qquad \int_0^1 g(x)\ dx \ = 4$$

Circle your answer.

13. (7 pts) Find the area of the region between the curves

$$y = x^3 - x^2 \quad \text{and} \quad y = x^2$$

Circle your answer.

14. (8 pts)

$$D(q) = 450 - q^2 \qquad S(q) = 150 + 2q^2$$

are the demand and supply functions for a particular commodity. Specifically q units will be demanded (sold) at a price $p = D(q)$ dollars per unit, while q units will be supplied by producers when the price is $p = S(q)$ dollars per unit.

(a) Find the equilibrium point (q_e, p_e) where supply equals demand.

(b) Find the consumer's surplus at equilibrium. This is the total amount consumers are willing to spend minus what consumers have to spend.

Answers

1. $\pi \ln(2)$

2. $x = 2$

3. $t = 0$ is rel min, $t = -4$ is rel max

4. $(2, 5)$ rel min, $(-2, -3)$ rel max

5. $p = \sqrt{50}$

6. $n = 10$

7. $10000\, e^{-.5}$

8. $f'(x) = 2^{-x} - x2^{-x} \ln(2)$

9. $f(0) = 0$
increasing for $x > -1$
decreasing for $x < -1$
global min at $x = -1$ and horizontal tangent
concave up for $x > -2$
concave down for $x < -2$
inflection point at $x = -2$
limit at ∞ is ∞
limit at $-\infty$ is 0^- x-axis is an asymptote

10. $y = \ln(x) + \frac{2}{x} + 3$

11. $2x + 1 - \ln|2x + 1| + C$

12. -2

13. $\frac{4}{3}$

14. $(10, 350)$ and $CS = 666\frac{2}{3}$