No books, no notes, no calculators, no cell phones, no pagers, no electronic devices of any kind. However you can bring to the exam

one 8.5 by 11

cheat sheet with anything you want written or printed on both sides.

Put your answer in the box provided if there is a box. Show every step of your solution. Erase or mark out anything false in your work. Simplify your answers as much as you can. It may not be sufficient to have a right answer to get full credit. Some graders may be stricter than others.

Exam and answers will be posted shortly after the exam: www.math.wisc.edu/∼miller

Name____________________________________

Circle the number (321-333) of your discussion section and hand in to your TA:

321	7:45	Kumar, Rohini
322	8:50	Kumar, Rohini
323	9:55	Mantilla Soler, Guillermo Arturo
324	11:00	Wang, Bing
325	12:05p	Wang, Bing
326	12:05p	Potluri, Vijaya Kranthi
327	1:20p	Mantilla Soler, Guillermo Arturo
328	1:20p	Umarji, Pallavi Anand
329	2:25p	Yin, Weidong
330	2:25p	Umarji, Pallavi Anand
331	3:30p	Yin, Weidong
332	2:25p	Christodoulopoulou, Konstantina
333	3:30p	Christodoulopoulou, Konstantina

Bing Wang's students hand in to Myoungjean Bae.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
13	10	
14	10	
15	10	
Total	150	

1. (10 pts)
Find the number c that makes $f(x)$ continuous for every x:

$$f(x) = \begin{cases} \frac{(1-x)^2}{1-x^2} & \text{if } x \neq 1 \\ c & \text{if } x = 1 \end{cases}$$

Put your answer
in the box.
Show work below.

2. (10 pts) Suppose you open an account paying 12% nominal annual interest compounded continuously. How much should you deposit to ensure that there will be $ 5,000 in the account after 6 months?

Put your answer

in the box.

Show work below.

3. (10 pts) Find the equation for the tangent line to the curve

$$xy^2 + 2y + 3x = 6$$

at the point $(1, 1)$.

Put your answer
in the box.
Show work below.

4. (10 pts) Find the inflection points of the function $f(x) = e^{-x^2}$.

Put your answer
in the box.
Show work below.

5. (10 pts) Suppose

$$G(t) = t^4 e^t + \int_0^t x^4 e^x \, dx$$

What is $G'(t)$?

Put your answer in the box. Show work below.

6. (10 pts)
Use the midpoint rule with $n = 2$ to approximate the integral

$$\int_0^1 (1 + x^2)\, dx.$$

Find the computed value C, the true value T of the integral, and the error, E.

Put your answer
in the box.
Show work below.

7. (10 pts)
Find the unique solution for the initial value problem:

$$\frac{dy}{dx} = \frac{y+1}{x-1} \quad \text{and} \quad y(0) = 1$$

Put your answer
in the box.
Show work below.

8. (10 pts) Park rangers have been studying the population dynamics of a the rabbits in a small park. They calculate that the logistic equation

$$\frac{dy}{dt} = ry - ky^2$$

is a good model for the number of rabbits $y(t)$ at time t if they take the damping factor $k = .003$ and the annual growth rate is $r = .51$. Suppose $y(0)$ is not zero and find the longterm number of rabbits in the park, i.e.,

$$\lim_{t\to\infty} y(t)$$

Put your answer
in the box.
Show work below.

9. (10 pts) Compute the following improper integral or show that it diverges:

$$\int_1^\infty xe^{-x^2}\,dx$$

Put your answer

in the box.

Show work below.

10. (10 pts) Sketch the level curves $z_0 = 1, 2, 3$ and the cross sections $x_0 = 2$ and $y_0 = 0$ for the surface

$$z = x^2 + y^2$$

11. (10 pts) Find all first-order partial derivatives for the function

$$f(x, y, z) = xy^2e^z$$

12. (10 pts) Find all critical points for the function

$$f(x,y) = x^2 + 3y^2 + 3y - 2x - 5$$

and determine whether each is a local minimum, local maximum, or saddle point.

Put your answer in the box. Show work below.	

13. (10 pts) Find the least squares line for the data:

$$(0,1) \quad (1,2) \quad (2,5)$$

Put your answer
in the box.
Show work below.

14. (10 pts) The function $f(x, y, z) = x^2 + 2y^2 + z^2$ has a minimum subject to the constraint $x - y + 2z = 2$. Find it by the method of Lagrange multipliers.

Put your answer in the box. Show work below.	

15. (10 pts)
Fill-in the blanks with the best possible answer:

1. A limit

$$\lim_{x \to 0} \frac{f(x)}{g(x)}$$

is an indeterminate form, if

$$\frac{\lim_{x\to 0} f(x)}{\lim_{x\to 0} g(x)} = \frac{0}{\underline{0}} \qquad \text{or} \qquad \frac{\lim_{x\to 0} f(x)}{\lim_{x\to 0} g(x)} = \frac{\infty}{\underline{\infty}}$$

If $f(x)$ is a continuous function on the interval $[a, b]$, $f(a) < 0$, and $f(b) > 0$, then for some c in (a, b) we must have that $\underline{f(c) = 0}$.

2. The following limit equals

$$\lim_{n \to \infty} (1 + \frac{1}{n})^n = \underline{e}.$$

3. The derivative of a function $f(x)$ is a new function $f'(x)$ that gives the slope of the tangent line to the graph at the point x. It is defined by the following limit:

$$f'(x) = \underline{\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

If a function f is differentiable, then it must be continuous.

4. If $f'(x) > 0$ for all x in an interval (a, b), then the function $f(x)$ is increasing on the interval.

The second derivative test says that if $f'(c) = 0$ and $\underline{f''(c) < 0}$, then f has a local maximum at c.

Suppose x_0 is in the interval (a, b) and for every x in the interval (a, b) we have that $f(x) \leq f(x_0)$, then we say that $f(x_0)$ is the global maximum of the function f in the interval.

5. An antiderivative of a given function $f(x)$ is a function $F(x)$ such that

$$F'(x) = f(x)$$

The Fundamental Theorem of Calculus states that if $f(x)$ is continuous on the interval $[a, b]$ and $F(x)$ is an antiderivative of $f(x)$, then

$$\int_a^b f(x)\, dx = F(b) - F(a)$$

If a function f is continuous on the interval $[a, b]$, then the integral $\int_a^b f(x)\, dx$ exists, i.e., the Riemann sums approach a finite limit.

6. Suppose $f(x)$ and $g(x)$ are continuous functions on the interval $[a, \infty)$, $0 \leq f(x) \leq g(x)$ for all x in $[a, \infty)$, and $\int_a^\infty f(x)\, dx$ diverges. Then $\int_a^\infty g(x)$ diverges

7. To find the maximum or minimum of $w = f(x, y, z)$, we look for all points (x, y, z) such that

$$\frac{\partial w}{\partial x} = 0 \text{ and } \frac{\partial w}{\partial y} = 0 \text{ and } \frac{\partial w}{\partial z} = 0$$

Answers

1. $c = 0$

2. $5000e^{-.06}$

3. $y = -x + 2$

4. $x = \pm\sqrt{\frac{1}{2}}$

5. $4t^3e^t + 2t^4e^t$

6. $T = \frac{64}{48}$, $C = \frac{63}{48}$, $E = \pm\frac{1}{48}$

7. $y = -2x + 1$

8. 170

9. $\frac{1}{2}e^{-1}$

10. circles $x^2 + y^2 = 1$, $x^2 + y^2 = 2$, $x^2 + y^2 = 3$
parbola $z = y^2 + 4$
parbola $z = x^2$.

11. $f_x = y^2e^z$, $f_y = 2xye^z$, $f_z = xy^2e^z$

12. $(1, -\frac{1}{2})$ is a local minimum.

13. $L(x) = 2x + \frac{2}{3}$

14. $(\frac{4}{11}, -\frac{2}{11}, \frac{8}{11})$ where f has the value $\frac{88}{121}$

15. See problem.