Show all work. Simplify your answers as much as you can. No books, no notes, no calculators, no cell phones, no pagers, no electronic devices of any kind. Use the last two pages of the exam for scratch paper or ask a TA for extra scratch paper. Exam and answers will be posted shortly after the exam: www.math.wisc.edu/∼miller

Name_____

Circle the number (321-333) of your discussion section and hand in to your TA:

321	7:45	TR	B329 VAN VLECK	Kumar, Rohini
322	8:50	TR	6322 SOC SCI	Kumar, Rohini
323	9:55	TR	348 BIRGE	Mantilla Soler, Guillermo Arturo
324	11:00	TR	1333 Sterling	Wang, Bing
325	12:05p	TR	348 BIRGE	Wang, Bing
326	12:05p	TR	B337 VAN VLECK	Potluri, Vijaya Kranthi
327	1:20p	TR	B135 VAN VLECK	Mantilla Soler, Guillermo Arturo
328	1:20p	TR	B123 VAN VLECK	Umarji, Pallavi Anand
329	2:25p	TR	B211 VAN VLECK	Yin, Weidong
330	2:25p	TR	B115 VAN VLECK	Umarji, Pallavi Anand
331	3:30p	TR	B135 VAN VLECK	Yin, Weidong
332	2:25p	TR	223 INGRAHAM	Christodoulopoulou, Konstantina
333	3:30p	TR	B115 VAN VLECK	Christodoulopoulou, Konstantina

Page	Points	Score
1	14	
2	14	
3	14	
4	14	
5	14	
6	14	
7	14	
8	2	
Total	100	

1. (7 pts) Assume that $\lim_{x\to 0} f(x) = 5$ and $\lim_{x\to 0} g(x) = -1$ find the limit:

$$\lim_{x\to 0} \frac{xf(x) - g(x)}{f(x) - g(x)}$$

2. (7 pts) Find the limit:

$$\lim_{x\to -\infty} \frac{x^3}{1 - x^2}$$

3. (7 pts) Determine where the function f is continuous and where it is discontinuous.

$$f(x) = \begin{cases} \frac{x^2-9}{x-3} & \text{if } x < 3 \\ \\ 6 & \text{if } x \geq 3 \end{cases}$$

4. (7 pts) Suppose b is a positive number and u is a fixed number for which it is known that

$$b^u = 5$$

What is b^{2u}?

5. (7 pts) Suppose you put \$2000 dollars in an account paying an annual nominal interest rate of 3.6% compounded continuously. How much will you have in the account at the end of 6 months?

6. (7 pts) Suppose that B and a are numbers such that $\log_2(B) = a$. Write the logarithm below as a function of a.

$$\log_2(\sqrt[3]{4B})$$

7. (7 pts) Rewrite x^y as an expression using the natural logarithm and the natural exponentiation function. (Suppose your calculator has no button for computing x^y but does have buttons for $\ln(x)$ and e^x. How could you use these to compute x^y?)

8. (7 pts) Find the equation of line tangent to the curve $y = x^2$ at the point $(2, 4)$.

9. (7 pts) Suppose that $f(x)$ is a differentiable function whose graph goes through the point $(1, 2)$ and whose tangent line at that point has the equation $x + y = 3$. Without computing, find each of the following limits:

(a) $\lim_{h\to 1} f(h)$

(b) $\lim_{\triangle x\to 0} \frac{f(1+\triangle x)-2}{\triangle x}$

(c) $\lim_{x\to 1} \frac{f(x)-2}{x-1}$

10. (7 pts) A miniature water rocket is launched straight upward from the surface of the earth. Its height s in feet at time t in seconds is given by the formula:

$$s = -16t^2 + 40t$$

Find the height and velocity of the rocket at the end of 1 second.

11. (7 pts) A real estate broker charges a commission of 7% percent on any house sold for \$100,000 or less. On sales that exceed \$100,000 the broker charges a flat fee of \$5,000 plus 3% of the sale price. Let x be the amount of a given sale and $f(x)$ the brokerage fee.

(a) Write a (multi-line) formula for $f(x)$ and sketch the graph.

(b) At what points, if any, is $f(x)$ discontinuous? At what points, if any, does $f(x)$ fail to be differentiable?

12. (7 pts) Find the derivative $\frac{dy}{dx}$ of

$$y = \frac{\ln(x) + 2}{3}$$

13. (7 pts) Find the derivative $\frac{dy}{dx}$ of

$$y = \frac{x^2 + 1}{1 + e^x}$$

14. (7 pts) Find the derivative $\frac{dy}{dx}$ of

$$y = (2x^4 - x^3 + 1)^7$$

15. (2 pts) Circle either True or False:

(a) For any real valued function f on the real line, if f is continuous at a point a, then f must be differentiable at the point a. True False

(b) There is a real valued function f defined for every real number which is continuous at every point but has no derivative at any point. True False

Answers

1. $1/6$

2. ∞

3. The function f is continuous at every point on the real line.

4. 25

5. The amount A in the account after t years satisfies

$$A = 2000e^{.036t}$$

Since 6 months is a half of a year you will have $2000e^{(.036)(1/2)} = 2000e^{.018}$

6. $\log_2(\sqrt[3]{4B}) = \log_2((4B)^{\frac{1}{3}}) = \frac{1}{3}\log_2(4B) = \frac{1}{3}(\log_2(4) + \log_2(B) = \frac{1}{3}(2 + a)$

7. $e^{y\ln(x)}$

8. $y = 4x - 4$.

9. (a) Since f is continuous this is $f(1) = 2$. (b) and (c) are equivalent definitions of $f'(1)$. Since the slope of the tangent line is -1, these are both -1.

10. The velocity is $\frac{ds}{dt} = -32t + 40$ at $t = 1$ we have that $s = 24$ and $\frac{ds}{dt} = 8$.

11. f is discontinuous at $x_0 = 100,000$ because

$$\lim_{x\to x_0^-} f(x) = \lim_{x\to x_0^-} (.07)x = 7000$$

$$\lim_{x\to x_0^+} f(x) = \lim_{x\to x_0^+} (.05)x + 5000 = 8000$$

Since differentiable implies continuous is not differentiable at x_0. Everywhere else it is differentiable and continuous. (Ignore $x \leq 0$.)

$$f(x) = \begin{cases} (.07)x & \text{if } x \leq 100000 \\ (.05)x + 5000 & \text{if } x > 100000 \end{cases}$$

12. $\frac{1}{3x}$

13. $\frac{(2x)(1+e^x)-(x^2+1)e^x}{(1+e^x)^2}$

14. $7(2x^4 - x^3 + 1)^6(8x^3 - 3x^2)$

15. (a) False (b) True