

No calculators.
No note or index cards.
Show all work.

Name_____

Problem	Points	Score
1	20	
2	25	
3	25	
4	20	
5	25	
6	20	
7	25	
8	20	
9	20	
Total	200	

1. (20 pts) A committee of 5 persons is to be formed from a group of 120 students. 80 of these students are Freshmen and 40 of them are Sophomores. How many possible committees are there in which there are more Sophomores than Freshmen but at least one Freshman?

2. (25 pts) In a random group of 5 people what is the probability that at least two have the same birthday?

3. (25 pts) Every day Professor NoodleBrain tosses a deck of cards one at a time at his hat trying to land the cards inside. After many years of practice the Professor has found that on average he lands about 13 of the 52 cards in his hat.

(a) What is the probability, p , that after tossing all 52 cards that at least 3 of the 4 aces lands in the hat?

(b) If he repeats this for 10 days in a row, what is the probability that on exactly 4 of the 10 days he gets at least 3 of the 4 aces in the hat? (You may express your answer in terms of p where p is from part (a).)

4. (20 pts) The axioms for a probability, $Pr(E)$ for $E \subseteq S$, are the following:
- (1) $0 \leq Pr(E) \leq 1$ for each $E \subseteq S$,
 - (2) $Pr(S) = 1$, and
 - (3) if E_1 and E_2 are disjoint subsets of S , then $Pr(E_1 \cup E_2) = Pr(E_1) + Pr(E_2)$.

Using these axioms deduce the formula

$$Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$$

where A and B are any subsets of S .

5. (25 pts) Consider the set of solutions of the system of equations

$$\begin{array}{rcrcrcrcrcl} x & + & 2y & + & z & = & 6 \\ 2x & + & y & + & z & = & 4 \\ ax & + & 3y & + & 2z & = & b \end{array}$$

Where a and b are constants.

(a) Give an example of an a and b such that this system has infinitely many solutions.

$a =$

$b =$

(b) Give an example of a and b such that this system has no solution.

$a =$

$b =$

6. (20 pts) Find a 3 by 3 matrix B so that $ABA = I$ where I is the 3 by 3 identity matrix and

$$A = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 0 & -1 \\ 1 & -1 & -1 \end{bmatrix} \quad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad B =$$

7. (25 pts) On January 1, 1998 you deposited \$3000 in a savings account which pays 4% compounded quarterly. On July 1, 1998 you withdrew \$1000. On July 1, 1999 how much will you have in this account?

8. (20 pts) On January 1, 2000 you buy a home for \$150,000. After paying a down payment of \$30,000 the remaining amount is borrowed from a bank at 7 % compounded monthly for 30 years. The first monthly payment is paid on January 1, 2000. What is the amount of each monthly payment?

9. (20 pts) You wish to save money for your child's college education. You contribute 2000 each year to a savings account which pays 8 % annual interest. Your first contribution is made on January 1, 2000, the second on January 1, 2001, and so on, making the same contribution each January. You make your last contribution on January 1, 2009. At that time, how much will your child have in this savings account?

Answers

1. $\binom{40}{4} \times \binom{80}{1} + \binom{40}{3} \times \binom{80}{2}$

2. $1 - \frac{365 \times 364 \times 363 \times 362 \times 361}{365^5}$

3. (a) $p = (\frac{1}{4})^4 + 4 \times (\frac{1}{4})^3 \times (\frac{3}{4})$

(b) $\binom{10}{4} p^4 (1-p)^6$

4. See Exam 2 answers.

5. (a) $a = 3 \quad b = 10$

(b) $a = 3 \quad b$ anything except 10.

6. $B = A^{-1} \times A^{-1}$

$$A^{-1} = \begin{bmatrix} -\frac{1}{2} & -1 & \frac{1}{2} \\ -1 & -1 & 0 \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{bmatrix}$$

$$B = \begin{bmatrix} 1\frac{1}{2} & 1\frac{1}{2} & -\frac{1}{2} \\ 1\frac{1}{2} & 2 & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

7. $(3000(1.01)^2 - 1000)(1.01)^4$

8. $\frac{120000}{y}$ where $y = \frac{1-x^{360}}{1-x}$ and $x = \frac{1}{1+i}$ and $i = \frac{.07}{12}$.

9. $2000(\frac{1-x^{10}}{1-x})$ where $x = 1.08$.