

No calculators. No note or index cards.

Show all work. Explain your answers.

In general, little credit will be given for answers alone.

You do not need to simplify fractions.

Name_____

TA: Winston Yang

Circle the time of your discussion section:

Tues 8:50

Tues 9:55

Thurs 8:50

Thurs 9:55

Problem	Points	Score
1	16	
2	16	
3	16	
4	18	
5	16	
6	18	
Total	100	

1. (16 %) Consider the set of solutions of the system of equations

$$ax - 2y = 6$$

$$x + y = b$$

Where a and b are constants. The following problems may have more than one correct solution. You should only give one, but explain why it is correct.

- (a) Give an example of an a and b such that this system has no solution.

$a =$

$b =$

- (b) Give an example of a and b such that this system has infinitely many solutions.

$a =$

$b =$

2. (16 %) The following system has a unique solution. Find it and plug it back into the system to verify that it is correct.

$$x + y + z + w = 10$$

$$-y + z + w = 5$$

$$3y + 2z + w = 16$$

$$2x + 2y + 2z = 12$$

$$x =$$

$$y =$$

$$z =$$

$$w =$$

3. (16 %) Calculate the following. (If the dimensions are incorrect, say that it is “undefined”.)

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 1 & 2 \\ 1 & -1 \end{bmatrix} =$$

$$\begin{bmatrix} 0 & 1 & 3 \\ -1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} =$$

$$\begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} \begin{bmatrix} 0 & 1 & 3 \end{bmatrix} =$$

4. (18 %) Find a 3 by 3 matrix B so that $A(B + I) = I$ where I is the 3 by 3 identity matrix and

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & -3 \\ 1 & 0 & 2 \end{bmatrix} \quad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad B =$$

5. (16 %) Formulate the following as a linear programming problem. Set up a table for the data. Identify the variables, the constraints, and the objective function. Maximize or minimize?

Do not attempt to solve the problem.

A farmer owns a **300** acre farm and can plant any combination of three crops. Crop A requires 1 man-day of labor and \$10 of capital for each acre planted. Crop B requires 4 man-days of labor and \$20 of capital for each acre planted. Crop C requires 3 man-days of labor and \$15 of capital for each acre planted. Crop A produces \$40 of revenue for each acre planted, Crop B \$60, and Crop C \$50. The farmer has \$3300 of capital and 485 man-days of labor available for the year. What is the optimal planting strategy?

Set up but do not solve. Labor is supplied by the farmer and his family and so costs nothing. Profit is revenue minus cost.

6. (18 %) Graph the region F determined by the following 5 inequalities. Determine all the corner points of F and shade the region F .

$$x \geq 0$$

$$y \geq 0$$

$$x + 3y \geq 6$$

$$x + y \geq 4$$

$$3x + y \geq 6$$

Find the minimum of $z = x + 2y$ and the maximum of $z = x + 2y$ in the region F . If one or the other doesn't exist, explain why.

Answers

1.

(a) $a = -2, b = 0$ (or any other pair with $a = -2$ and $b \neq -3$)

$$-2x - 2y = 6$$

$$x + y = 0$$

implies

$$x + y = -3$$

$$x + y = 0$$

which has no solutions since it implies $0 = 3$.(b) $a = -2, b = -3$ since this makes both equations the same.

$$-2x - 2y = 6$$

$$x + y = -3$$

2. The augmented matrix is

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 10 \\ 0 & -1 & 1 & 1 & 5 \\ 0 & 3 & 2 & 1 & 16 \\ 2 & 2 & 2 & 0 & 12 \end{array} \right] \quad \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 10 \\ 0 & -1 & 1 & 1 & 5 \\ 0 & 3 & 2 & 1 & 16 \\ 0 & 0 & 0 & -2 & -8 \end{array} \right] \quad \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 10 \\ 0 & -1 & 1 & 1 & 5 \\ 0 & 0 & 5 & 4 & 31 \\ 0 & 0 & 0 & -2 & -8 \end{array} \right]$$

$$(-2 \times \text{row}_1) + \text{row}_4 \Rightarrow \text{row}_4$$

$$(3 \times \text{row}_2) + \text{row}_3 \Rightarrow \text{row}_3$$

Backsubstituting gives:

$$\begin{array}{rclcl} x + y + z + w & = & 10 & \Rightarrow x = 10 - y - z - w = 10 - 2 - 3 - 4 = 1 & \Rightarrow x = 1 \\ -y + z + w & = & 5 & \Rightarrow -y = 5 - z - w = 5 - 3 - 4 = -2 & \Rightarrow y = 2 \\ 5z + 4w & = & 31 & \Rightarrow 5z = 31 - 4w = 31 - 16 = 15 & \Rightarrow z = 3 \\ -2w & = & -8 & & \Rightarrow w = 4 \end{array}$$

3.

$$\left[\begin{array}{cc} -1 & 1 \\ 0 & 2 \\ 2 & 0 \end{array} \right] \quad \left[\begin{array}{c} 4 \\ 2 \\ 0 \end{array} \right] \quad \left[\begin{array}{ccc} 0 & -1 & -3 \\ 0 & 1 & 3 \\ 0 & 2 & 6 \end{array} \right]$$

4. $A(B + I) = I$ so $B + I = A^{-1}$ and $B = A^{-1} - I$. To find A^{-1} :

$$\left[\begin{array}{ccc|ccc} 0 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & -3 & 0 & 1 & 0 \\ 1 & 0 & 2 & 0 & 0 & 1 \end{array} \right]$$

swap rows

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & -3 & 0 & 1 & 0 \end{array} \right]$$

$(row_1 + row_3) \Rightarrow row_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 1 \end{array} \right]$$

$(row_3 \times 2 + row_1) \Rightarrow row_1$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 2 & 3 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 1 \end{array} \right]$$

$(row_3 \times -1) \Rightarrow row_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 2 & 3 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 \end{array} \right]$$

$$B = A^{-1} - I = \begin{bmatrix} -1 & 2 & 3 \\ 1 & -1 & 0 \\ 0 & -1 & -2 \end{bmatrix}$$

5. We use the following table:

	Labor	Capital	Revenue	Profit
Crop A	1	10	40	30
Crop B	4	20	60	40
Crop C	3	15	50	35
	485	3300		

Letting x be the number of acres of Crop A, y Crop B, and z Crop C. Then we seek to maximize the profit

$$P = 30x + 40y + 35z$$

subject to the constraints:

$$x, y, z \geq 0$$

$$x + y + z \leq 300$$

$$x + 4y + 3z \leq 485$$

$$10x + 20y + 15z \leq 3300$$

6. There are four corner points: $(0, 6)$, $(1, 3)$, $(3, 1)$, $(6, 0)$. The region F is unbounded and contains points (x, y) which are arbitrarily large, so $z = x + 2y$ has no maximum. Its minimum is at $(3, 1)$ where $z = 5$.