

Practice Final Exam A. Miller Fall 98 Math 210

Most of these problems are from: “Finite Mathematics”, Barnett, Ziegler, and Byleen, Prentice Hall. If you are having trouble reading Chapter 9 of the text, go to the library and look for any book with “Finite Mathematics” in its title (try QA 39.2). Most of them have a similar chapter.

Final exam in 180 Science on Tues Dec 22 at 5:05 pm.

1. Grandparents deposit 6000 dollars into a grandchild’s account toward a college education. How much money will be in the account 17 years from now if the account earns 9 percent compounded monthly?
2. A car salesperson tells you that you can buy the car you are looking at for 3000 dollars down and 200 dollars a month for 48 months. If interest is 14 percent compounded monthly, what is the selling price of the car and how much interest will you pay during the 48 months?
3. A scholarship committee wishes to establish a sinking fund that will pay 1500 dollars a quarter to a student for 2 years. How much should they deposit now into an account that pays 8 percent compounded quarterly in order to make the scholarship payments starting 3 months from now?
4. You unexpectedly inherit 10,000 dollars just after you have made the 72nd monthly payment on a 30 year mortgage of 60,000 at 10.25 percent compounded monthly. Determine the relative merits of using the inheritance to reduce the principal of the loan or to buy a certificate of deposit paying 8.75 percent compounded monthly.
 - (a) If you buy a certificate of deposit with the 10,000, how much will it be worth when you make your last (i.e. 360th) mortgage payment?
 - (b) If you use the 10,000 to reduce the principal, how many remaining mortgage payments must you make? If you invest the remaining payments at 8.75 percent compounded monthly, how much would you have at the time of the 360th payment?

5. Find the inverse of

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -1 \\ 2 & 3 & 0 \end{bmatrix}$$

6. An investor has 60000 to invest in a CD and a mutual fund. The CD yields 5 percent. The mutual fund yields an average of 9 percent and requires a minimum investment of 10000. The investor requires that at least twice as much be invested in the CD as in mutual funds. How much should be invested in CDs and how much should be invested in mutual funds to maximize the return? What is the maximum return?

7. Suppose 6 female and 5 male have been successfully screened for 5 positions. If the five positions are filled at random from the 11 finalists, what is the probability of selecting:

- (a) 3 females and 2 males?
- (b) 4 females and 1 male?
- (c) 5 females?
- (d) At least 4 females?

8. In a random sample of 1000 people, it is found that 7 percent have a liver ailment. Of those who have a liver ailment, 40 percent are heavy drinkers, 50 percent are moderate drinkers, and 10 percent are nondrinkers. Of those who do not have a liver ailment, 10 percent heavy drinkers 70 percent are moderate drinkers, and 20 percent are nondrinkers. If a person is chosen at random and it is found that he or she is a heavy drinker, what is the probability that person has a liver ailment? What is the probability for a nondrinker?

9. Home ownership in the United States declined during the 1980's ending a 40 year period of growth. The 1980 census reported that 45.1 percent of the households in a particular city were homeowners and the remainder were renters. During the next decade 12 percent of the homeowners became renters and the rest remained homeowners. Likewise, 5 percent of the renters became homeowners and the rest continued to rent.

- (a) Draw the transition graph.
- (b) Write the appropriate transition matrix.
- (c) According to this transition matrix, what percentage of households were homeowners in 1990?
- (d) If the transition matrix remains the same, what percentage of households will be homeowners in 2000?
- (e) Under the same assumption, in the distant future what percentage will be homeowners?

10. A fair coin is flipped.

- If it comes up H, toss it a second time, otherwise stop.
- If the second toss is H, toss a third time, otherwise stop.

The random variable X = the total number of heads obtained. The random variable Y = the number of times you toss the coin.

What are the possible values of X ? What are the possible values of Y ? What is the expected value of X ? What is the expected value of Y ? What is the expected value of $X + Y$?

11. (a) How many words can be made up from the letters from ABCDEFG? (Every letter must be used exactly once.)

(b) How many words can be made up from the letters from ABCDEFG, if we require that the letters A and B may **not** occur next to each other?

12. Formulate the following as a linear programming problem. Be sure and identify the variables, the constraints, and the objective function. **Do not attempt to solve the problem.**

In an experiment on conditioning, a psychologist works with four kinds of rodents: white mice, black mice, white rats, and hairless rats. He uses three types of Skinner (conditioning) boxes. A box might contain a maze or it might contain a mechanism which shocks the rodent when it tries to drink.

Each mouse spends 10 minutes per day in Box A and 20 minutes per day in Box B.

Each rat spends 15 minutes in Box A and 5 minutes per day in Box B.

In addition to this time each white rodent spends 10 minutes in Box C and each black rodent spends 30 minutes in Box C. (Hairless rats spend no time in Box C.)

The experimental design requires that at least as many mice as rats be used.

The psychologist can only work with one rodent at time. She plans to spend 8 hours (480 minutes) on Monday working with box A, 8 hours on Tuesday working with box B, and 8 hours on Wednesday working with box C.

What is the largest number of rodents which can be experimented on?

Answers

1. $6000(1 + \frac{.09}{12})^{12 \times 17} = 27551$

2. Assume the first payment is on the day of closing. Selling price of Car is

$$10404 = 3000 + 200\left(\frac{1 - x^{48}}{1 - x}\right)$$

where $x = \frac{1}{1+i}$ and $i = \frac{.14}{12}$. Payout = $3000 + 200 \times 48$.

Interest = Payout - Selling price = 2196.

3. $i = \frac{.08}{4}$ and $p = 1500$. They will need

$$\frac{p}{1+i} + \frac{p}{(1+i)^2} + \frac{p}{(1+i)^3} + \dots + \frac{p}{(1+i)^8}$$

which equals

$$\frac{p}{1+i}(1 + x + x^2 + \dots + x^7)$$

where $x = \frac{1}{1+i}$ and which is by the geometric series formula

$$\frac{p}{1+i}(1 + x + x^2 + \dots + x^7) = \frac{p}{1+i} \left(\frac{1 - x^8}{1 - x} \right) = 10988.$$

4. The certificate would be worth 81042 when the 360th payment is made.

$$81042 = 10000 \left(1 + \frac{.0875}{12} \right)^{288}$$

The mortgage payment satisfies

$$60000 = \frac{p}{1+i} \left(\frac{1 - x^{360}}{1 - x} \right) \text{ where } i = \frac{.1025}{12} \text{ and } x = \frac{1}{1+i}$$

if we assume that the first payment is at the end of the first month. Solving for p gives us, $p = 538$ (monthly payment). The principal on the loan after $72 = 360 - 288$ payments is

$$58039 = \frac{p}{1+i} \left(\frac{1 - x^{288}}{1 - x} \right)$$

After reducing 58039 to 48039 find n so that

$$48039 = \frac{p}{1+i} \left(\frac{1-x^n}{1-x} \right)$$

The value of n is closest to the integer 169 so the loan would be paid off after the 241=169+72 payments. If the remaining 119 = 360 - 241 monthly payments to the bank were instead invested at 8.75 percent compounded monthly, at the time of the original loan's 360th payment they would be worth

$$101375 = 538(1+j)^{118} + 538(1+j)^{117} + \dots + 538(1+j) + 538 = 538 \left(\frac{1-y^{119}}{1-y} \right)$$

where $j = \frac{.0875}{12}$ and $y = 1 + j$.

It is always better to pay off the mortgage if the mortgage rate is higher than the CD rate, even if taxes are taken into consideration.

5.

$$\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$$

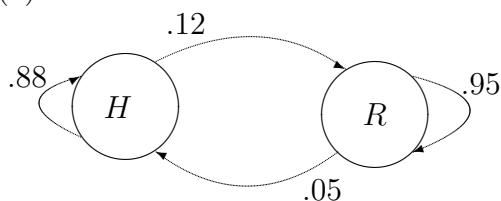
6. 40000 in CD and 20000 in Mutual Funds, returns 3800.

7. (a) .433 (b) .162 (c) .013 (d) .175

8. .231 .036

9.

(a)



(b)

$$\begin{bmatrix} .88 & .12 \\ .05 & .95 \end{bmatrix}$$

(c) 42.433 percent (d) 40.219 percent (e) 29.412 percent

10. X has possible values 0,1,2,3. Y has possible values 1,2,3. $E(X) = \frac{7}{8}$,
 $E(Y) = 1\frac{3}{4}$, $E(X + Y) = E(X) + E(Y)$

11. (a) $7!$ (b) $7! - (2 \times 6!)$

12. Let w = number of white mice, b = number of black mice, r =
number of black rats, h = number of hairless rats,

Maximize the number of

Rodents = $w + b + r + h$

subject to:

$$\begin{aligned} w, b, r, h &\geq 0 \\ 10(w + b) + 15(r + h) &\leq 480 \\ 20(w + b) + 5(r + h) &\leq 480 \\ 10(w + r) + 30b &\leq 480 \\ w + b &\geq r + h \end{aligned}$$