

No notes, no books, no calculators, no cell phones, no pagers, no electronic devices of any kind.

Name_____

Circle your TAs name:

Song Sun

(Boyd) Chalermpong Worawannotai

Hand in your exam to your TA.

Problem	Points	Score
1	13	
2	13	
3	13	
4	13	
5	13	
6	13	
7	13	
8	13	
9	13	
10	13	
11	13	
12	13	
13	4	
Total	160	

Solutions will be posted shortly after the exam:
www.math.wisc.edu/~miller/m210

1. (13 pts) (a) Translate the sentence below into a symbolic logical expression using the statement letters p , q , and r and logical symbols. State what each letter (p , q , and r) stand for.

If Obama dies or becomes incapacitated, then Biden becomes president.

- (b) Construct the truth table for the symbolic logical expression you found in part (a).

2. (13 pts) A club has 10 members. In how many ways are there to choose a slate of four officers consisting of a president, vice-president, secretary, and treasurer?

3. (13 pts) Find an equation for the line which contains the two points $(1, 4)$ and $(-2, 3)$.

4. (13 pts) (a) If the nominal annual interest rate is 12 percent compounded monthly, what is the effective quarterly rate?

(b) If the nominal annual interest rate is 12 percent compounded quarterly, what is the effective annual rate?

(c) If the effective annual rate is 8 percent, what is the effective quarterly rate?

5. (13 pts) Harriet goes to a movie each Saturday night. She always sees one of two kinds of movie: either a comedy movie or a romantic movie. If she sees a comedy, the next week she is equally likely to see either another comedy movie or a romantic movie. If she sees a romantic movie, the next week she is twice as likely to see a comedy movie as a romantic movie.

(a) Identify the appropriate states and draw a transition diagram or graph.

(b) Find the transition matrix.

(c) If Harriet saw a romantic movie on December 6, what is the probability she will see a comedy movie on December 20?

6. (13 pts) Juan and Maria opened a savings account on September 1 with an initial deposit of 3000 Pesos¹. The bank pays a (special introductory) monthly effective rate of 2 percent for the first month (September), but thereafter only 1 percent monthly. On December 1 Juan goes to a Cajero automático² and finds out that their bank account contains only 2200 Pesos. It turns out that Maria has withdrawn some money from their account on November 1 to buy a new purse. How much did she withdraw?

¹Mexican dollars

²ATM Automated Teller Machine

7. (13 pts) The chairman surveyed the 45 professors in the math department and asked each to identify exactly two of the following three important departmental concerns: parking (low cost and within fifty feet of the math building), office equipment (cappuccino makers and massage chairs), and chalk reliability (non-disintegrating and multicolored). The result of his survey was:

29 identified parking

33 identified office equipment

28 identified chalk reliability

How many of the math professors identified parking and chalk as their two concerns?

8. (13 pts) The proportion of individuals in Madison whose salary is more than 50000 is 60 percent. About 70 percent of those whose salary is more than 50000 are college graduates, while only 20 percent of those whose salary is less than or equal to 50000 are college graduates. What is the probability that a randomly chosen college graduate in Madison has a salary of more than 50000?

9. (13 pts) A pair of dice is rolled and the maximum of the two numbers³ is noted as the random variable X . Find the expected value of X .

³Note, for example, that the maximum of rolling two threes is three.

10. (13 pts) Find $A + A^{-1}$ where:

$$A = \begin{bmatrix} 4 & -3 \\ 3 & -2 \end{bmatrix}$$

11. (13 pts) A furniture company makes two types of furniture: chairs and stools. The production process is divided into two distinct operations: carpentry and upholstery. The manufacture of each chair requires: 6 man-hours of carpentry and 2 man-hours to upholster. The manufacture of each stool requires: 3 man-hours of carpentry and 6 man-hours to upholster. The company has available each day a maximum of 48 man-hours for carpentry and 36 man-hours for upholstery. If the profit on each chair is 4 dollars and on each stool is 3 dollars, how many chairs and stools should be produced each day to maximize the profit?

12. (13 pts) Li-Tsung and Man-Yin are to become engaged. Li wants to buy Man a diamond ring for 12000 Yuan⁴. He plans to pay for it with a sizable down-payment and to borrow the remainder. He wants to make 16 quarterly payments of 500 Yuan each, payable at the end of each quarter, in addition to a down-payment at the beginning of the first quarter when he purchases the ring. The jeweler will give him a loan at the quarterly effective interest rate of 2 percent. How much is Li's down-payment? Use the geometric series formula⁵ to express your answer in closed form.

⁴Chinese dollars

⁵Geometric series formula:

$$1 + x + x^2 + \cdots + x^n = \frac{1 - x^{n+1}}{1 - x}$$

The expression to the right of the equal sign is the closed form for that on the left.

13. (4 pts) For each of the statements below, circle the best answer:

T for True or F for False

1. (*T* or *F*)

Nominal annual interest is the same as effective annual interest, if the compounding period is exactly one year.

2. (*T* or *F*)

If a system of linear equations has more than one solution, then it must have infinitely many solutions.

3. (*T* or *F*)

Two symbolic logical expressions are logical equivalent, if the last column of their truth tables are the same.

4. (*T* or *F*)

If X and Y are random variable on a probability space, then

$$E(X - 2Y) = E(X) - 2E(Y)$$

Answers

1. p is Obama dies, q is Obama is incapacitated, r is Biden becomes president.

p	q	r	$(p \vee q) \rightarrow r$
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	T
F	F	F	T

2. 5040

3. $y = (1/3)x + (11/3)$

4.

(a) $i = (1.01)^3 - 1$

(b) $j = (1.03)^4 - 1$

(c) $k = (1.08)^{1/4} - 1$

5. (a) The states are C and R . The two outgoing arrows from state C are both labeled $1/2$, the arrow from R to C is labeled $2/3$ and the arrow from R to R is labeled $1/3$.

(b)

$$M = \begin{bmatrix} 1/2 & 1/2 \\ 2/3 & 1/3 \end{bmatrix}$$

(c) $5/9$

6. 912.38 Pesos

7. 12

8. .84

9. $161/36$

10.

$$A + A^{-1} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

11. 6 chairs and 4 stools.

12. 5211.14 Yuan

13. All are true