No notes, no books, no calculators, no cell phones, no pagers, no electronic devices of any kind.

Name____

Circle your TAs name:

Song Sun (Boyd) Chalermpong Worawannotai

Hand in your exam to your TA.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

Solutions will be posted shortly after the exam:
www.math.wisc.edu/~miller/m210

1. (10 pts) A random variable X has the following probability density function:

k	Pr[X=k]
1	.1
2	.2
3	.7

Find the expected value of X, $\mu = E(X)$.

2. (10 pts) You have one card marked 5, two cards marked 10, and one card marked 15. Two cards are chosen randomly and simultaneously and their numbers are noted. A random variable X is defined to be the sum of the numbers on the cards.

What are the possible values of X? (Do not repeat values.)

Find the density function of X.

3. (10 pts) A Bernoulli process consists of 100 trials with the probability of success on each trial of $p = .2$. A random variable X consists of the number of successes in the 100 trials.

What is the expected value of X?

What is the standard deviation of X?

4. (10 pts) Solve the system and graph the two lines:

$$x + 4y = 20$$

$$3x + 2y = 30$$

5. (10 pts) Let F be the set of all points (x, y) which satisfy all of the three inequalities:

$$\begin{aligned} x + y &\geq 6 \\ y &\leq x - 4 \\ y &\geq 0 \end{aligned}$$

Graph the set F, shade it, and find the coordinates of its corner points.

6. (10 pts) For the following system of equations:

$$3x - y + 6z = 19$$
$$x + y + 2z = 9$$

(a) find the augmented matrix of the system;

(b) show that $x = 1$, $y = 2$, $z = 3$ is a solution of the system;

(c) show that there are no solutions with $y = 0$.

7. (10 pts) Let

$$P = \begin{bmatrix} 0 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

(a) Find P^2

(b) Find P^4

8. (10 pts) Find the inverse (if it exists) of the matrix A:

$$A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 \\ 1 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

9. (10 pts) The Bait Shop sells three kinds of bait packages, Type A, Type B, and Type C packages. The packages contain worms, minnows, and grasshoppers in the amounts shown in the following table.

Type	Worms	Minnows	Grasshoppers
A	25	10	10
B	15	5	15
C	10	10	25

The profit (per package) is 2 dollar for type A packages, 3 dollar for type B, and 5 dollars for type C. There are 500 worms, 400 minnows, and 600 grasshoppers available. How many packages of each type should be made to maximize profit?

Formulate this linear maximization problem but do **not** solve it. Identify
(a) the variables (what does each stand for),
(b) the function to be maximized, and
(c) the constraints.

10. (10 pts)
Let F be the set of points (x, y) satisfying all of the inequalities:

$$\begin{aligned} x &\geq 1 \\ y &\geq 1 \\ x &\leq 5 \\ y &\leq 5 \\ x + y &\leq 8 \end{aligned}$$

(a) Find the minimum value of $z = 2x + y$ in the set F.

(b) Find the maximum value of $z = 2x + y$ in the set F.

Answers

1. $\mu = 2.6$

2.

k	Pr[X=k]
15	1/3
20	1/3
25	1/3

4. $(8, 3)$ plus a graph which shows the lines and some of the intercepts.

5. The region is infinite with corner points $(5, 1)$ and $(6, 0)$.

6. (a)

$$\begin{bmatrix} 3 & -1 & 6 & 19 \\ 1 & 1 & 2 & 9 \end{bmatrix}$$

(b) $3(1) - (2) + 6(3) = 19$ and $(1) + (2) + 2(3) = 9$
(c) Putting $y = 0$ means that x and z must satisfy

$$3x + 6z = 19$$

$$x + 2z = 9$$

But adding (-3) times the second equation to the first, gives us the equation $0 = -8$ which has no solutions.

7. (a)

$$P^2 = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

(b)

$$P^4 = P^2P^2 = \begin{bmatrix} 5 & -4 & 1 \\ -4 & 6 & -3 \\ 1 & -3 & 2 \end{bmatrix}$$

8. $A^{-1} =$

$$\begin{bmatrix} 1 & 0 & 0 & -1 \\ 1 & 1 & 0 & -1 \\ -1 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

9. (a) x is the number of packages of type A to be made. y is the number of packages of type B to be made. z is the number of packages of type C to be made.

(b) Maximize the profit $P = 2x + 3y + 5z$

(c) Subject to the constraints:

$x \geq 0$

$y \geq 0$

$z \geq 0$

$25x + 15y + 10z \leq 500$

$15x + 5y + 15z \leq 400$

$10x + 10y + 25z \leq 600$

10. (a) minimum value of z is 3 at the point $(1, 1)$.

(a) maximum value of z is 13 at the point $(5, 3)$.