

**Show all work.**

**You do not have to simplify your answer unless asked to do so.**

**Circle your answer.**

No books and no notes. You may use a calculator but no other electronic devices.

Name \_\_\_\_\_

Circle your Discussion Section:

|     |   |              |      |          |
|-----|---|--------------|------|----------|
| 343 | T | 12:05--12:55 | 1412 | STERLING |
| 344 | R | 12:05--12:55 | 1327 | STERLING |
| 345 | T | 13:20--14:10 | 1327 | STERLING |
| 346 | R | 13:20--14:10 | 1327 | STERLING |

| Problem | Points | Score |
|---------|--------|-------|
| 1       | 12     |       |
| 2       | 12     |       |
| 3       | 12     |       |
| 4       | 12     |       |
| 5       | 12     |       |
| 6       | 10     |       |
| Total   | 70     |       |

Solutions will be posted shortly after the exam: [www.math.wisc.edu/~miller/m210](http://www.math.wisc.edu/~miller/m210)

1. (12 pts) The three Axioms for a Probability Measure:

A probability measure assigns to each event  $E$  of a sample space  $S$  a number denoted by  $Pr[E]$  and called the probability of  $E$ . This assignment must satisfy three axioms.

1.  $Pr(E') = 1 - Pr(E)$  for any event  $E$ .
2.  $Pr(E_1 \cap E_2) = Pr(E_1)Pr(E_2)$  if  $E_1$  and  $E_2$  are independent events
3.  $Pr(E_1|E_2) = \frac{Pr(E_1 \cap E_2)}{Pr(E_2)}$  for any events  $E_1$  and  $E_2$
4.  $Pr(E_1 \cup E_2) = Pr(E_1) + Pr(E_2)$  if  $E_1$  and  $E_2$  are disjoint events
5.  $E(X + Y) = E(X) + E(Y)$
6.  $Pr(\emptyset) = 0$
7.  $Pr(S) = 1$
8.  $Pr(E_1 \cup E_2) = Pr(E_1) + Pr(E_2) - Pr(E_1 \cap E_2)$  for any events  $E_1$  and  $E_2$
9.  $0 \leq Pr(E) \leq 1$  for any event  $E$
10.  $Var(X) = E(X - \mu)^2$

Choose exactly 3:

2. (12 pts) There are 3 blue and 5 green balls in a box. Balls are selected at random one after another without replacement, until a green ball is selected. Let  $X$  be a random variable whose value is the number of balls drawn.

Find the density function of  $X$  and the expected value of  $X$ .

Simplify your answers.

3. (12 pts) Find the total amount of interest (in dollars) earned on an investment of \$7,000 with a nominal annual interest rate of 6% compounded monthly over a period of two years. You do not have to simplify your answer. Circle your answer.

4. (12 pts) The Wisconsin Badgers Turnip Company decides to raise funds for expansion by issuing zero coupon notes which are payable in 3 years. The notes sell for \$2000 and are redeemable 3 years later for \$2300. If such a note is purchased and held to maturity, what is the effective annual percentage yield to the investor?

You do not have to simplify your answer. Circle your answer.

5. (12 pts) Jebediah is buying a new car. He has \$3000 in cash and can borrow the rest of the money needed to buy the car from his Credit Union at a nominal annual interest rate of 12% compounded monthly. He purchases an \$11,000 car by putting down the cash and financing the rest for 8 equal monthly payments, each made at the beginning of the month, and starting on the day he buys the car. What is the amount of each payment?

You do not have to simplify your answer. Circle your answer.

6. (10 pts) Let  $U = \{1, 2, 3, 4, 5, 6, 7\}$  be the universal set and  $A, B, C$  subsets of  $U$ . Circle True if the formula is true for all  $A, B, C$  subsets of  $U$  and Circle False otherwise.

1.  $n(A \cap B) = n(A) + n(B) - n(A \cup B)$  *True* *False*
2.  $n(A \times B) = n(A) + n(B)$  *True* *False*
3.  $A \cup (B \cup C) = (A \cup B) \cup C$  *True* *False*
4.  $A' \cup B = (A \cap B)'$  *True* *False*
5. If  $n(A) = 5$  and  $n(B) = 4$ , then  $A \cap B \neq \emptyset$ . *True* *False*
6.  $(A \cup B)' = A' \cap B'$  *True* *False*
7.  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$  *True* *False*
8.  $n(A) = 7 - n(A')$  *True* *False*
9.  $A \subseteq B$  iff  $A' \subseteq B'$  *True* *False*
10.  $C'' = C$  *True* *False*

## Answers

1. 4,7,9

2.

| k | Pr(X=k) |
|---|---------|
| 1 | 13/56   |
| 2 | 15/56   |
| 3 | 5/56    |
| 4 | 1/56    |

$$E(X) = 1.5$$

3. 890.12

4. 4.8%

5. 1035.17

6. TFTTT TFTFT