Show all work.
Simplify your answers.
Circle your answer.

No notes, no books, no calculator, no cell phones, no pagers, no electronic devices.

Name______________________________________

Circle your Discussion Section:

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DIS 343 12:05p T B329 VAN VLECK
DIS 344 12:05p R B321 VAN VLECK
DIS 345  1:20p T  595 VAN HISE
DIS 346  1:20p R 3401 STERLING
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Problem	Points	Score
1	8	
2	9	
3	8	
4	8	
5	9	
Total	42	

Solutions will be posted shortly after the exam: www.math.wisc.edu/~miller/m210

1. (8 pts)
Prove the geometric series formula:

$$1 + x + x^2 + \cdots + x^n = \frac{1 - x^{n+1}}{1 - x}$$

(for $x \neq 1$)

2. (9 pts)

(a) What is the effective quarterly rate j which corresponds to a nominal annual rate of 8 percent?

(b) What is the effective annual rate k corresponding to a nominal annual rate of 8 percent which is compounded quarterly?

(c) Suppose P=$1000 is put in a savings account which earns a nominal annual interest rate of 8 percent compounded quarterly. After 18 months how much is in the account?

3. (8 pts)

Sam buys a car from his sister Sally for \$3000. He agrees to pay her \$1000 two years from now and the remaining amount A three years from now. Assuming an effective annual interest rate of 6 percent, how much is A?

4. (8 pts)

Harry bought 100 shares in the Inca Lost Gold Mine Corporation (ILGM) for \$90 per share on Nov 15, 2000. On April 1, 2001 the stock price rises to \$120 per share and the company makes a two for one split so that each share is now worth \$60. On Nov 15, 2002 rumors hit the market that there is no lost Inca Gold and the stock price of ILGM plunges to \$10 per share. However, the company finds that tourists will pay a lot of money to be taken out into the Yucatan Jungle to search for the Inca Lost Gold Mine. The price of each share of ILGM rises steadily to \$80 on Nov 15, 2004 at which time Harry sells all his shares of ILGM stock.

Did Harry lose money or make money on his investment? What was the effective annual yield (or loss) on his investment?

5. (9 pts) A loan is paid back in three annual payments of p dollars made at the end of each year. The effective annual interest rate is i.

(a) What is the principle or amount borrowed B? Assume that it is borrowed at the beginning of the first year. (Express B in terms of p and i.)

(b) What is the total amount of interest I (in dollars) paid thru the course of the loan? (Express I in terms of B, p, and i.)

(c) Each payment p in a loan is part interest and part reduction in the principle. What part I_1, I_2, I_3 of each of the three payments is interest?

Answers

1. Put $S = 1 + x + x^2 + \cdots + x^n$. Then

$$xS = x(1 + x + x^2 + \cdots + x^n) = x + x^2 + x^3 \cdots + x^n + x^{n+1}.$$

If we subtract we get:

$$S - xS = 1 - x^{n+1}.$$

Hence $(1-x)S = 1 - x^{n+1}$ or $S = \frac{1-x^{n+1}}{1-x}$.

2.
(a) $j = .08/4 = .02$
(b) $(1+k) = (1+j)^4$ or $k = (1.02)^4 - 1$.
(c) 18 months is a year and a half or 6 quarters. Hence the answer is $1000(1.02)^6$.

3.

$$3000 = \frac{1000}{(1.06)^2} + \frac{A}{(1.06)^3}$$

Hence

$$A = (1.06)^3 \left(3000 - \frac{1000}{(1.06)^2} \right)$$

4. Harry made money. On Nov 15, 2000 he paid \$9000 for his 100 shares. The stock split 2 for 1 which means that each stock holder doubles his number of shares. On Nov 15, 2004 he sold his 200 shares for \$16000. The yield is the effective annual interest rate which when compounded would result in the same amount of gain. His annual yield i satisfies:

$$16000 = 9000(1+i)^4.$$

Hence

$$i = (16/9)^{1/4} - 1.$$

As far as I know there is no mathematical difference between
yield = rate of return = interest rate
The four year rate of return k would satisfy

$$9000(1+k) = 16000$$

since in this case there would be one four year period of time. Hence if we set $P = 9000$ and $Q = 16000$ then

$$P(1+k) = Q$$

$$1 + k = \frac{Q}{P}$$

$$k = \frac{Q}{P} - 1 = \frac{Q-P}{P}$$

$Q - P = 7000$ is the gain and $P = 9000$ is the amount invested. Hence

$$k = 7/9 = .7777\ldots \approx 78\%$$

The nominal annual rate of return j would satisfy

$$4j = k.$$

Hence $j \approx 19\frac{1}{2}\%$. The effective annual rate of return i should satisfy

$$(1+i)^4 = (1+k)$$

Which is the same i as above. Using a calculator we get that i is approximately $14\frac{1}{2}\%$.

5. (a)

$$B = \frac{p}{(1+i)} + \frac{p}{(1+i)^2} + \frac{p}{(1+i)^3}$$

(b) Everything paid is either principle or interest so

$$3p = B + I \text{ or } I = 3p - B$$

(c) At the beginning of the second year there are only two payments left, so the remaining principle is

$$C = \frac{p}{(1+i)} + \frac{p}{(1+i)^2}$$

Thus the first payment has reduced the principle by

$$B - C = \frac{p}{(1+i)^3}$$

The first payment $p = I_1 + (B - C)$ hence

$$I_1 = p - \frac{p}{(1+i)^3}.$$

Similarly at the beginning of the third year there is only one payment left, so the remaining principle is

$$D = \frac{p}{(1+i)}$$

so the second payment has reduced the principle by

$$C - D = \frac{p}{(1+i)^2}.$$

Thus $p = I_2 + (C - D)$ hence

$$I_2 = p - \frac{p}{(1+i)^2}.$$

Finally at the beginning of the fourth year there is no priniciple left and hence the third payment reduces the principle by $D - 0 = \frac{p}{(1+i)}$ and so $p = I_3 + (D - 0)$ and so

$$I_3 = p - \frac{p}{(1+i)}.$$