Show all work.
Simplify your answers.
Circle your answer.

No notes, no books, no calculator, no cell phones, no pagers, no electronic devices.

Name_____

Circle your Discussion Section:

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DIS 332   9:55 R 395 VAN HISE
DIS 331   9:55 T  52 BASCOM
DIS 335 12:05p T 123 INGRAHAM
DIS 336 12:05p R 114 INGRAHAM
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Problem	Points	Score
1	32	
2	32	
3	32	
4	32	
5	32	
6	20	
7	20	
8	20	
9	20	
Total	240	

Solutions will be posted shortly after the exam: www.math.wisc.edu/∼miller/m210

1. (32 pts) Let A and C be sets with $A \subseteq C$. Suppose that $n(A) = 4$ and $n(C) = 12$. In how many different ways can you select B so that $A \subseteq B$, $B \subseteq C$, and $n(B) = 6$?

2. (32 pts) At another ballroom dance lesson there are the same 4 girls and 5 boys. Recall that one of the boys is Mortimer who has a secret crush on one of the girls, Hortense. At the end of the lesson the instructor puts slips of paper with the boy's names in a bucket and slips of paper with the girl's names in another bucket. He randomly draws one boy's name and one girl's name and has the couple demonstrate the Waltz. He then returns the two slips to their respective buckets and repeats the drawing procedure and has this couple demonstrate the Rumba.

(a) What is the probability that Mortimer and Hortense demonstrate both dances?

(b) What is the probability that Mortimer and Hortense demonstrate at least one of the two dances?

3. (32 pts) A farmer wishs to mix two feeds for her dairy herd to provide the necessary calories, vitamins, and minerals at minimum cost. The amounts of each found in feeds F_1 and F_2 are as follows. F_1 provides 3 units per pound of calories, 1 unit per pound of vitamins, and 1 unit per pound of minerals. F_2 provides 4 units per pound of calories, 1 unit per pound of vitamins, and 5 units per pound of minerals. The minimal requirements per cow per day are 340 calorie units, 100 vitamin units, and 150 mineral units. F_1 costs 3 dollars per pound and F_2 costs 5 dollars per pound. If the farmer wants to minimize the total cost of her feed mix, how many pounds of each feed should she use?

4. (32 pts) Find all solutions of the systems of equations:

$$x - y + 3z = 4$$
$$2x + y = 2$$
$$x + z = 2$$

5. (32 pts) A hotel chain is considering the construction of a resort hotel at a certain beach. On sunny days in the summer season, the hotel can expect to fill its anticipated 200-room facility. On rainy days during the summer season, the experience of existing hotels shows that a 50 per cent capacity can be expected, while on cloudy days a 75 per cent capacity can be expected. The Weather Bureau records indicated that during the summer season at that location, there is a 70 per cent probability of clear sunny weather, 10 per cent probability of rainy days and 20 per cent probability of cloudy days. Let X be a random variable denoting the number of rooms rented on one day in the summer. What is the expected value of X?

6. (20 pts) Consider a Markov chain whose transition matrix is

$$A = \begin{bmatrix} .3 & 0 & .7 \\ .2 & .1 & .7 \\ .4 & .6 & 0 \end{bmatrix}$$

In which state is the sytem least likely to be in the long run and what is the probability that it is one of the other two states?

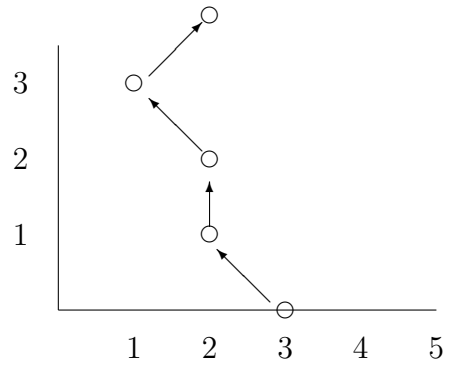
7. (20 pts) (Drunkard's walk). A drunk is stumbling along a sidwalk. He randomly lurches forward at each step. With equal probability he lurches either one unit straight forward, or one unit forward and one unit left, or one unit forward and one unit right. You can visualize his progress by thinking of him as walking along the integer points (m, n) in the plane. If he is at the point (m, n), then he lurches upward to one of the points $(m-1, n+1)$, $(m, n+1)$, or $(m+1, n+1)$. See example walk below left.

The y-axis is a wall. If he is at any point $(1, n)$ and tries to move to $(0, n+1)$ at the next step he bounces off the wall and moves to $(1, n+1)$ instead, so he is twice as likely to move to $(1, n+1)$ as to $(2, n+1)$.

The line $x = 5$ is the gutter, if he falls into it he can't get out.

(a) Draw a Markov graph using the states $1, 2, 3, 4, 5$ representing the x-coordinate of his position. Find the transistion matrix A.

(b) What is the probability that starting at the point $(3, 0)$ that he takes 100 steps and does not fall into the gutter? (You may express your answer in terms of A^n for appropriately chosen n, please do not explicitely calculate A^{100}.)



8. (20 pts) A car is advertised in the newspaper as follows: You receive a cash bonus B of \$3275 together with your car (i.e., the reverse of a down payment). In return for which you must pay 30 monthly payments p of \$398 each. The first payment is to be made exactly six months after you receive the car and cash bonus. If the monthly interest rate for car loans is $i = .004825$ per month, what is the cost C of the car? This is the same as asking what would you expect to pay for the car in cash on the day of the sale, instead of the bonus plus payments deal being offered.

9. (20 pts)

Mrs Jones wants to buy her son Max an IPOD for a present. She plans to pay for it by making two deposits in her savings account. The first deposit she makes on Nov 1, 2003 and the second on Dec 1, 2003. The second deposit will be twice as large as the first. On Feb 1, 2004 she pays for the $300 IPOD by withdrawing the money from her account. Her savings account earns 1/2% a month in interest. What is the size of the smaller deposit?

Answers

1. 28

2. (a) 1/400 (b) 39/400

3. Minimize $z = 3x + 5y$ subject to

$$x, y \geq 0$$

$$3x + 4y \geq 340$$

$$x + y \geq 100$$

$$x + 5y \geq 150$$

The critical points are $(0, 100), (60, 40), (100, 10), (150, 0)$. The minimum occurs at $x = 100$ pounds of F_1 and $y = 10$ pounds of F_2.

4. There are infinitely many solutions. One description of this set is: z anything, $y = 2z - 2$, and $x = 2 - z$.

5. $E(X) = 200(.7) + 100(.1) + 150(.2) = 180$.

6. The stable vector is $(\frac{16}{51}, \frac{14}{51}, \frac{21}{51})$ so the answer is $\frac{37}{51}$

7. (a)

$$A = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) $1 - entry_{3,5}(A^{100})$

8. The buyer gets the bonus B and the car or equivalently the cash value of the car C. The seller gets the present values of the 30 payments. In a fair deal each one gets the same, otherwise they will not be willing to make the exchange. Hence

$$B + C = \frac{p}{(1+i)^6} + \frac{p}{(1+i)^7} + \cdots + \frac{p}{(1+i)^{35}}$$

where $B = 3275$, $p = 398$, and $i = .004825$. So

$$C = \frac{p}{(1+i)^6} + \frac{p}{(1+i)^7} + \cdots + \frac{p}{(1+i)^{35}} - B$$

Using the geometric series formula this can also be written:

$$C = \frac{p}{(1+i)^6}\left(\frac{1-\frac{1}{(1+i)^{30}}}{1-\frac{1}{(1+i)}}\right) - B$$

9. Let $i = .005$ and let p be the amount of the first deposit. The amount of the second deposit is $2p$. The amount in the savings account on Feb 1 is $p(1+i)^3+2p(1+i)^2$. Hence

$$300 = p(1+i)^3 + 2p(1+i)^2$$

and so

$$p = \frac{300}{(1.005)^3 + 2(1.005)^2}$$