

Show all work.

Simplify your answers.

Circle your answer.

No notes, no books, no calculator, no cell phones, no pagers, no electronic devices.

Name \_\_\_\_\_

Circle your Discussion Section:

DIS 332 9:55 R 395 VAN HISE

DIS 331 9:55 T 52 BASCOM

DIS 335 12:05p T 123 INGRAHAM

DIS 336 12:05p R 114 INGRAHAM

Problem	Points	Score
1	12	
2	12	
3	12	
4	12	
5	12	
6	12	
7	12	
8	12	
9	12	
10	12	
Total	120	

Solutions will be posted shortly after the exam: [www.math.wisc.edu/~miller/m210](http://www.math.wisc.edu/~miller/m210)

Note: This exam counts the same as the first exam - the total on the first exam will be doubled.

1. (12 pts) The Jacks, Queens, Kings, and Aces (16 cards) are removed from an ordinary deck of cards. These 16 cards are repeatedly shuffled and then the top two cards are dealt out on a table face up in front of you. For each Ace you are paid \$2 and for each King you are paid \$1.

The random variable  $X$  is defined to be the total paid to you. Find the probability density function of  $X$ .

2. (12 pts) A coin is weighted so that the probability of Heads is  $2/3$ . The coin is flipped 27 times. The random variable  $X$  is the number of Heads which occur. Find  $\mu = E[X]$  (the expected value of  $X$ ),  $\nu = Var[X]$  (the variance of  $X$ ), and  $\sigma$  (the standard deviation of  $X$ ).

3. (12 pts) Find the slope, x-intercept, y-intercept, and graph the line with equation:

$$x = 2y + 6$$

4. (12 pts) For what numbers  $a$  and  $b$  is the system

$$ax + y = b$$

$$x - by = -3$$

satisfied by  $x = 1$  and  $y = 2$ ?

5. (12 pts) Find all solutions of the systems of equations:

$$x + y - z = 2$$

$$x - y + 3z = 4$$

6. (12 pts) True or False.

1.  $A(B + C) = AB + AC$  for all  $6 \times 6$  matrices  $A, B$  and  $C$ .

Circle one: True False

2.  $A(BC) = (AB)C$  for all  $10 \times 10$  matrices  $A, B$  and  $C$ .

Circle one: True False

3.  $(B + C)A = AB + AC$  for all  $7 \times 7$  matrices  $A, B$  and  $C$ .

Circle one: True False

4. Every  $2 \times 2$  matrix except the zero matrix has an inverse.

Circle one: True False

5. If  $A$  is a  $2 \times 3$  matrix, then  $A$  has 2 columns and 3 rows.

Circle one: True False

6.  $A + B$  is defined if and only if  $A$  and  $B$  have the same dimension.

Circle one: True False

7. (12 pts) Find the inverse of  $A$  or say that it does not exist.

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$



8. (12 pts) In the following problem formulate a linear programming problem. Be sure and identify the variables, the constraints, and the objective function.

Do **not** solve the problem.

The government of Wisconsin is planning to inoculate the population against the flu. There are 400 doctors, 800 nurses, and 2000 technicians available to carry out this plan. There are 10 million people who need to be inoculated.

An inoculation team can consist of either one doctor, three nurses, and 6 technicians (type 1), or one doctor, two nurses, and 10 technicians (type 2), or no doctors, 4 nurses, and 12 technicians (type 3). On average a type 1 team can inoculate 12 people per hour, type 2 teams can inoculate 13 people per hour, and type 3 teams can inoculate 10 people per hour. How many teams of each type should be formed to maximize the number of inoculations per hour?

9. (12 pts) A set of points is described by the inequalities:

$$x \geq 0$$

$$y \geq 1$$

$$x + y \leq 4$$

$$2x + 3y \geq 6$$

Graph this region in the plane. Find the vertices of the polygon described by the inequalities.

10. (12 pts) Find the maximum of  $z = 2x + y$  subject to

$$y \leq x$$

$$3x + 2y \leq 30$$

$$6y \geq x$$

## Answers

1.

$$\text{Prob}(X = 4) = 6/120 \text{ (AA)}$$

$$\text{Prob}(X = 3) = 16/120 \text{ (AK)}$$

$$\text{Prob}(X = 2) = 38/120 \text{ (KK or A?)}$$

$$\text{Prob}(X = 1) = 32/120 \text{ (K?)}$$

$$\text{Prob}(X = 0) = 28/120 \text{ (??)}$$

2.  $\mu = 18, \nu = 6, \sigma = \sqrt{6}$

3. slope is  $1/2$ , intercepts are  $(0, -3)$  and  $(6, 0)$ 

4.  $a = 0, b = 2$

5.  $z$  anything,  $y = -1 + 2z$  and  $x = 3 - z$ 

6. TTFFFT

7. This matrix does not have an inverse.

8. Maximize  $I = 12x + 13y + 10z$ 

subject to

$$x, y, z \geq 0$$

$$x + y \leq 400$$

$$3x + 2y + 4z \leq 800$$

$$6x + 10y + 12z \leq 2000$$

where  $x$  is the number of type 1 teams,  $y$  is the number of type 2 teams, and  $z$  is the number of type 3 teams.

9.  $(0, 2), (0, 4), (3/2, 1), (3, 1)$

10.  $19 \frac{1}{2}$