

Metric properties of capacities

One of the classical problems in complex analysis is the Painlevé problem (1898) on removable singularities for bounded analytic functions. The removable sets are exactly the sets of zero analytic capacity (the concept of analytic capacity was introduced by Ahlfors in 1947). In 1992 Paramonov introduced a natural multi-dimensional analog of analytic capacity (Lipschitz harmonic capacity), which characterizes the removable sets for Lipschitz harmonic functions. In turn, this capacity is a particular case of the capacity $\gamma_{s,+}$ generated by vector-valued Riesz potentials (Tolsa, Volberg); $\gamma_{s,+}$ also is related to other capacities, for example to Riesz capacities in non-linear potential theory.

In the talk we consider certain metric properties of $\gamma_{s,+}$ (first of all, the connection between $\gamma_{s,+}$ and Hausdorff measure). Various Cantor sets demonstrate the sharpness of the results obtained, as well as the difference between capacities generated by potentials with vector-valued and with positive Riesz kernels.