We correct an error in our paper "Singular maximal functions and Radon transforms in L^1"m American Journal of Mathematics, 126 (2004), 607–647. The error is in the estimation of the measure of the exceptional set. The argument in the beginning of §5 of the paper should be replaced by the following. We thank Neal Bez for pointing out this error.

The estimate for the exceptional set

We shall prove the nontrivial estimate (3.20) for the maximal function, assuming again that the curvature assumption in the introduction is satisfied, and prove the inequality

$$\text{meas}\Big(\Big\{x : \sup_k \Big| \sum_{\substack{n,l \\ k\in(I_l^n)^*}} \mu_k^n * B_l^n \Big| > \alpha\Big\}\Big) \le \int \Phi(|f|/\alpha)dx \tag{5.1}$$

with $\Phi(t) = t\log\log(e^2+t)$.

We use the decomposition in Proposition 4.1 and form an additional exceptional set $\mathcal{O}_1$. To define it we set for $q\in\mathfrak{Q}_0$, $\kappa\in(I_l^n)^*$,

$$F_q^{n,l,\kappa}(x) = \sum_{\substack{r(w)\in I_l^n \\ r(w)<\kappa}} f_w^{n,\kappa}(x)\chi_q(\delta_{-\kappa}x). \tag{5.2}$$

and define

$$\mathcal{O}_1 = \bigcup_{n=1}^{\infty}\bigcup_{l\in\mathbb{Z}}\bigcup_{\kappa\in(I_l^n)^*}\bigcup_{q\in\mathfrak{Q}_0}\bigcup_{\substack{k\in(I_l^n)^* \\ k\le\kappa}} \text{supp }\big(\mu_k^n * F_q^{n,l,\kappa}\big); \tag{5.3}$$

moreover we define

$$\mathcal{O} = \mathcal{O}_1\cup\Omega^* \tag{5.4}$$

where Ω^* is as in (3.2).

To estimate the measure of $\mathcal{O}_1$ observe that supp $(\mu_k^n * F_q^{n,l,\kappa}) = \delta_\kappa\text{supp }(\mu_{k-\kappa}^n * [F_q^{n,l,\kappa}(\delta_\kappa\cdot)])$.

We claim that

$$\text{meas}\big(\text{supp }\big(\sum_{k\le\kappa}\mu_{k-\kappa} * [F_q^{n,l,\kappa}(\delta_\kappa\cdot)]\big)\big) \lesssim \Lambda_n[F_q^{n,l,\kappa}(\delta_\kappa\cdot)]$$

We need to improve Lemma 2.4

Lemma. *Let f be supported on a set of diameter $\le C_0$. Then*

$$\text{meas}(\text{supp }(\cup_{k\le 0}\mu_k^n * f)) \lesssim \Lambda_n(f).$$

with the implicit constants depending on C_0 and the diameter of the support of μ_0.

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Proof. We cover the set $\{x : f(x) \neq 0\}$ with cubes Q of sidelength $\geq 2^{-n}$ so that

$$\sum \ell(Q) \leq 2\Lambda_n(f).$$

Denote this collection of cubes by $\mathcal{Q}$.

We need to bound the measure of $\cup_{k\leq 0} \cup_Q (Q + \text{supp } \mu_k^n))$ by $C \sum_Q \ell(Q)$. Let $\mathcal{Q}_k^1$ be the subfamily of cubes which have sidelength $\geq 2^{ka}$, recall that $a > 0$ is less than the real part of all eigenvalues of P. Then we need to show

$$(*) \qquad \text{meas}(\cup_{k\leq 0} \cup_{Q\in\mathcal{Q}_k^1} (Q + \text{supp } \mu_k^n)) \lesssim \Lambda_n(f);$$

$$(**) \qquad \text{meas}(\cup_{k\leq 0} \cup_{Q\in\mathcal{Q}\setminus\mathcal{Q}_k^1} (Q + \text{supp } \mu_k^n))\Lambda_n(f).$$

Note that for $Q \in \mathcal{Q}_k^1$ the set $Q + \text{supp } \mu_k^n$ is contained in a fixed C-dilate of Q. Thus

$$\text{meas}(\cup_{k\leq 0} \cup_{Q\in\mathcal{Q}_k^1} (Q + \text{supp } \mu_k^n)) \lesssim \sum_Q |Q| \lesssim \sum_Q \ell(Q) \lesssim \Lambda_n(f)$$

thus (*).

Th show (**) we fix $k \leq 0$ and consider the set

$$Q + \text{supp } \mu_k^n = 2^{kP}(\text{supp } \mu_0^n + 2^{-kP}Q).$$

The set $2^{-kP}Q$ can be covered with $O(2^{-k(\tau-ad)})$ cubes q of sidelength $\ell(q) = 2^{-ka}\ell(Q)$ (≤ 1), or if, we insist on working with dyadic cubes we require $\ell(q) \approx 2^{-ka}\ell(Q)$. This uses again that a is smaller than the real part of every eigenvalue of P.

Thus

$$\begin{aligned}
&\text{meas}(Q + \text{supp } \mu_k^n)) = 2^{k\tau}\text{meas}(\text{supp } \mu_0^n + 2^{-kP}Q) \\
&\lesssim 2^{k\tau}\text{meas}(\sum_q \text{supp } \mu_0^n + q) \lesssim \sum_q 2^{k\tau}\text{meas}(\text{supp } \mu_0^n + q) \\
&\lesssim \sum_q 2^{k\tau}\ell(q) \lesssim 2^{-k(\tau-ad)}2^{k\tau}2^{-ka}\ell(Q) \\
&\lesssim 2^{ka(d-1)}\ell(Q)
\end{aligned}$$

and thus

$$\text{meas}(\cup_{k\leq 0} \cup_{Q\in\mathcal{Q}\setminus\mathcal{Q}_k^1} (Q + \text{supp } \mu_k^n)) \lesssim \sum_{k\leq 0} 2^{ka(d-1)} \sum_Q \ell(Q) \lesssim \Lambda_n(f). \quad \square$$

From (5.3.1) and (4.3) we get

$$
\begin{aligned}
\operatorname{meas}(\mathcal{O}_1) &\lesssim \sum_{n=1}^{\infty}\sum_{l\in\mathbb{Z}}\sum_{\kappa\in(I_l^n)^*} 2^{\kappa\tau}\operatorname{meas}\big(\operatorname{supp}\,\big(\sum_{k\le\kappa}\mu_{k-\kappa}*[F_q^{n,l,\kappa}(\delta_\kappa\cdot)]\big)\big)\\
&\lesssim \sum_{n=1}^{\infty}\sum_{l\in\mathbb{Z}}\sum_{\kappa\in(I_l^n)^*} 2^{\kappa\tau}\Lambda_n[F_q^{n,l,\kappa}(\delta_\kappa\cdot)]\\
&\lesssim \sum_{n=1}^{\infty}\sum_{l\in\mathbb{Z}}\sum_{\kappa\in(I_l^n)^*} 2^{\kappa\tau}\operatorname{meas}\big(\operatorname{supp}\,\big(\sum_{k\le\kappa}\mu_{k-\kappa}*[F_q^{n,l,\kappa}(\delta_\kappa\cdot)]\big)\big)\\
&\lesssim \sum_{n=1}^{\infty}\sum_{l\in\mathbb{Z}} 2^{\kappa\tau}\sum_{\kappa\in(I_l^n)^*}\alpha^{-1}\int F_q^{n,l,\kappa}(\delta_\kappa x)dx\\
&\lesssim \sum_{n=1}^{\infty}\sum_{l\in\mathbb{Z}}\sum_{\kappa\in(I_l^n)^*}\alpha^{-1}\int F_q^{n,l,\kappa}(x)dx\\
&\lesssim \sum_{q\in\mathfrak{Q}_0}\alpha^{-1}\int |F_q^{n,l,\kappa}(y)|dy \lesssim \sum_{n=1}^{\infty}\sum_{w}\alpha^{-1}\int |b_w^n(y)|dy \lesssim \alpha^{-1}\sum_{n=1}^{\infty}\sum_{w}\int |f_w^n(y)|dy\\
&\lesssim \alpha^{-1}\int |f(y)|dy \qquad (5.5)
\end{aligned}
$$

and the measure of $\mathcal{O} = \mathcal{O}_1 \cup \Omega^*$ satisfies the same estimate.