QUASILINEAR OPERATORS WITH NATURAL GROWTH TERMS

BENJAMIN J. JAYE

In this talk, we will describe some joint work with V. G. Maz'ya and I. E. Verbitsky, concerning homogeneous quasilinear differential operators. The model operator under consideration is:

$$\mathcal{L}(u) = -\Delta_p u - \sigma|u|^{p-2}u.$$

Here $\Delta_p = \mathrm{div}(|\nabla u|^{p-2}\nabla u)$ is the p-Laplacian operator, and σ is a signed measure, or more generally a distribution. We are primarily concerned with the operator $\mathcal{L}$ under conditions on σ where classical regularity results such as the Harnack inequality may fail.

Firstly, we will consider the connections between the existence of positive solutions of the homogeneous equation $\mathcal{L}(u) = 0$ with the problem of characterizing certain Sobolev inequalities with indefinite weight. Many of the results here are new in the case $p = 2$, where the operator $\mathcal{L}$ reduces to the Schrödinger operator.

Second, we will discuss an approach to studying the pointwise behavior of solutions to the Dirichlet problem:

$$\mathcal{L}(u) = \omega, \quad \inf_{x\in\mathbb{R}^n} u(x) = 0,$$

where ω is a nonnegative measure. This approach goes via the study of certain nonlinear integral equations.

Department of Mathematics, University of Missouri, Columbia, MO 65211, USA
E-mail address: bjjm93@mizzou.edu

Date: February 17, 2011.