DIMENSION INDEPENDENT BOUNDS FOR THE SPHERICAL MAXIMAL FUNCTION ON PRODUCTS OF FINITE GROUPS

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1. Abstract

The Hardy-Littlewood maximal operators (ball and cube) are perhaps the most fundamental in Euclidean Harmonic Analysis; celebrated results of Stein and Strömberg and Bourgain establish the following dimension-independent bounds:

Theorem 1. *For each $p > 1$ there exists an absolute constant A_p* independent of dimension *so that*

$$\|\mathcal{M}\|_{L^p(\mathbb{R}^N) \to L^p(\mathbb{R}^N)} \leq A_p.$$

Here, $\mathcal{M}$ is either Hardy-Littlewood maximal operator.

In this talk, we will extend these results to *spherical* maximal functions acting on cartesian products of cyclic groups equipped with the Hamming metric:

For $m \geq 2$, let $(\mathbb{Z}_m^N, |\cdot|)$ denote the group equipped with the l^0 (aka Hamming) metric,

$$|y| = \left|\big(y(1), \ldots, y(N)\big)\right| := \left|\{1 \leq i \leq N : y(i) \neq 0\}\right|,$$

define the L^1-normalized indicator of the r-sphere,

$$\sigma_r := \frac{1}{|\{|x| = r\}|} 1_{\{|x|=r\}}.$$

and the maximal function

$$M^N f(x) := \sup_{r \leq N} |\sigma_r * f|$$

acting on functions defined on $\mathbb{Z}_m^N$. Here, convolution is defined

$$\sum_{y \in \mathbb{Z}_m^N} f(x-y)\sigma_k(y),$$

with subtraction occuring bitwise $\mod m$.

Each maximal function is bounded by the operator $\Sigma^N f := \sum_{r \leq N} |\sigma_r * f|$, which has

$$\left\|\Sigma^N\right\|_{L^p(\mathbb{Z}_m^N) \to L^p(\mathbb{Z}_m^N)} \leq N + 1;$$

in particular, bounding M^N with the dimension of the group is not hard to do at all; we will use semi-group techniques and Fourier analysis to prove the following dimension-independent estimates:

Theorem 2. *For all $m \geq 2$ and $p > 1$, there exist absolute constants $C_{m,p}$ so that* for each N

$$\left\|M^N\right\|_{L^p(\mathbb{Z}_m^N) \to L^p(\mathbb{Z}_m^N)} \leq C_{m,p}.$$

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