DIOPHANTINE EQUATIONS IN THE PRIMES AND ERGODIC THEOREMS

Abstract. Let Q be a positive polynomial with integer coefficients in the n variables $\mathbf{x} = (x_1, ..., x_n)$, and denote the set of solutions to the equation $Q(\mathbf{x}) = \lambda$ by $r_Q(\lambda)$. Also let (Y, μ) be a probability space with a commuting family of measure preserving invertible transformations $T = (T_1, ..., T_n)$. Under a strong ergodicity condition of the family T, it is known that the averages

$$\frac{1}{r_Q(\lambda)} \sum_{Q(\mathbf{x})=\lambda} f(T^{x_1} ... T^{x_n} y),$$

with $f \in L^2(Y, \mu)$, converge to $\int_Y f d\mu$ in $L^2(Y, \mu)$ as $\lambda \to \infty$ through a collection of regular values. The pointwise analogue of this result is also known to hold. The talk will focus on showing results of a similar style, but with the added caveat that the x_i's are restricted to the set of primes.