MULTI-PARAMETER OSCILLATORY SINGULAR INTEGRALS WITH POLYNOMIAL PHASES

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ABSTRACT. Let $\Lambda = (\Lambda_1, \cdots, \Lambda_d)$ with $\Lambda_\nu \subset \mathbb{Z}_+^n$, and set $\mathcal{P}_\Lambda$ the family of polynomials:

$$\mathcal{P}_\Lambda = \left\{ P_\Lambda : P_\Lambda(t) = \left(\sum_{\mathfrak{m} \in \Lambda_1} c_{\mathfrak{m}}^1 \, t^{\mathfrak{m}}, \cdots, \sum_{\mathfrak{m} \in \Lambda_d} c_{\mathfrak{m}}^d \, t^{\mathfrak{m}} \right) \text{ with } \ t \in \mathbb{R}^n \right\}.$$

Given $P_\Lambda \in \mathcal{P}_\Lambda$, we consider a class of multi-parameter oscillatory singular integrals,

$$\mathcal{I}(P_\Lambda, \xi, r) = \text{p.v.} \int_{\prod [-r_j, r_j]} e^{i\xi \cdot P_\Lambda(t)} \frac{dt_1}{t_1} \cdots \frac{dt_n}{t_n} \ \text{ where } \ \xi \in \mathbb{R}^d, r \in \mathbb{R}_+^n.$$

When $n = 1$, $\mathcal{I}(P_\Lambda, \xi, r)$ for any $P_\Lambda \in \mathcal{P}_\Lambda$ is known to be bounded uniformly in ξ and r. However, when $n \geq 2$, the uniform boundedness depends on each indivisual polynomial P_Λ. In this talk, we fix Λ and find a necessary and sufficient condition on Λ such that

$$\text{for all } P_\Lambda \in \mathcal{P}_\Lambda, \ \sup_{\xi,\, r} |\mathcal{I}(P_\Lambda, \xi, r)| \leq C_{P_\Lambda} < \infty. \tag{0.1}$$

The condition is described by faces and their cones of polyhedra associated with Λ_ν's.

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2000 *Mathematics Subject Classification.* Primary 42B20, 42B25.

Key words and phrases. Multiple Hilbert transform, Newton polyhedron, Face, Cone, Oscillatory Singular Integral.