MADISON LECTURES IN HARMONIC ANALYSIS 2024

Schedule and Abstracts

Monday, May 13
Van Vleck Hall B102

9:00 Welcome

Morning Session

Chair: Jacob Fiedler (UW Madison)

9:05-9:55 Rodrigo Bañuelos
Norm estimates for discrete singular integrals

10:00-10:30 *Coffee break*

10:30-11:20 Polona Durcik
Quantitative norm convergence of triple ergodic averages for commuting transformations

11:30-12:20 Tuomas Hytönen
Convex body domination and other reduced inequalities for vector-valued functions

Afternoon Session

Chair: Lingxiao Zhang (Univ. of Connecticut)

3:00 - 4:00 p.m. Joris Roos
Isoperimetric and Poincaré inequalities on the Hamming cube

4:00 - 4:50 p.m. Malabika Pramanik
Numbers – are they normal?

Tuesday, May 14
Van Vleck Hall B102

Morning Session

Chair: Sarah Tammen (UW Madison)

9:00-9:50 Yumeng Ou
New improvement to Falconer's distance set problem

10:00-10:30 *Coffee break*

10:30-11:20 David Beltran
Endpoint sparse bounds

11:30-12:20 Jonathan Fraser
The Fourier spectrum and applications to exceptional sets

Afternoon Session

Chair: Terry Harris (UW Madison)

3:00 - 4:00 p.m. Xiumin Du
Weighted refined decoupling and Falconer distance set problem

4:00 - 4:50 p.m. Ruixiang Zhang
A new conjecture to unify Fourier restriction and Bochner-Riesz

Wednesday, May 15
Van Vleck Hall B102

First Morning Session

Chair: James Tautges (UW Madison)

9:00-9:50 Hong Wang
Furstenberg sets estimate in the plane

10:00-10:30 *Coffee break*

Second Morning Session

Chair: Jacob Denson (UW Madison)

10:30-11:20 Larry Guth
A new approach to bounding large values of Dirichlet polynomials.

11:30-12:20 Jonathan Bennett
Adjoint Brascamp–Lieb inequalities

No talks in the afternoon.

Thursday, May 16
Van Vleck Hall B102

Morning Session

Chair: Shengwen Gan (UW Madison)

9:00-9:50 Detlef Müller
An FIO approach to L^p bounds for wave equations associated to sub-Laplacians

10:00-10:30 *Coffee break*

10:30-11:20 Krystal Taylor
How to use projections to cover a fractal set by curves without wasting measure

11:30-12:20 Philip Gressman
Generalized sublevel set inequalities for differential forms

Afternoon Session

Chair: Amelia Stokolosa (UW Madison)

3:00 - 4:00 p.m. Jonathan Hickman
Variable coefficient L^p local smoothing

4:00 - 4:50 p.m. Ciprian Demeter
When the Schrödinger equation meets number theory

Friday, May 17

Van Vleck Hall B102

Morning Session

Chair: Jianhui Li (Northwestern University)

9:00-9:50 Tony Carbery
Radial weights in the Mizohata–Takeuchi conjecture

10:00-10:30 Coffee break

10:30-11:20 Azita Mayeli
Eigenvalue distribution of spatio-spectral limiting operators in higher dimensions

11:30-12:20 Christoph Thiele
Maximally modulated singular integrals on doubling metric measure spaces

Afternoon Session

Chair: Rajula Srivastava (HCM Bonn)

3:00 - 4:00 p.m. Joshua Zahl
Curve tangencies and maximal functions.

4:00 - 4:50 p.m. James Wright
Oscillatory integrals over locally compact fields: A unified theory.

List of Abstracts

Anthony Carbery
University of Edinburgh

Title: Radial weights in the Mizohata–Takeuchi conjecture

Abstract: After briefly discussing recent joint work with Marina Iliopoulou and Hong Wang and its relation to the decoupling literature, we revisit the case of radial weights in the Mizohata–Takeuchi conjecture and attempt to provide a more robust framework for their study. This is joint work with Marina Iliopoulou.

Azita Mayeli
City University of New York

Title: Eigenvalue distribution of spatio-spectral limiting operators in higher dimensions

Abstract: Let F, S be bounded measurable sets in $\mathbb{R}^d$. Let $P_F : L^2(\mathbb{R}^d) \to L^2(\mathbb{R}^d)$ be the orthogonal projection on the subspace of functions with compact support on F, and let $B_S : L^2(\mathbb{R}^d) \to L^2(\mathbb{R}^d)$ be the orthogonal projection on the subspace of functions with Fourier transforms having compact support on S. In this talk, I will report on the distributional estimates on the eigenvalue sequence $1 \geq \lambda_1(F,S) \geq \lambda_2(F,S) \geq \cdots > 0$ of the *spatio-spectral limiting operator* $B_S P_F B_S : L^2(\mathbb{R}^d) \to L^2(\mathbb{R}^d)$. The significance of such estimates lies in their diverse applications in wireless communications, medical imaging, signal processing, geophysics and astronomy.

More precisely, for suitable domains F and S, we prove that for any $\epsilon \in (0,1)$

$$\#\{k : \lambda_k(F,S) > \epsilon\} = (2\pi)^{-d}|F| \cdot |S| + \mathrm{Err}(F,S,\epsilon),$$

where the error term depends on $\mathcal{H}_{d-1}(\partial F)$ and $\mathcal{H}_{d-1}(\partial S)$, denoting the $(d-1)$-dimensional Hausdorff measures of the boundaries of F and S, respectively, and the geometric constants related to an Ahlfors regularity condition on the domain boundaries.

Our proof is based on the dyadic decomposition of both spatial and frequency domains, the decomposition techniques of operators, and the application of the results in my previous joint paper with Arie Israel (ACHA 2024) on the eigenvalues of spatio-spectral limiting operators associated to cubical domains.

This is joint work with Kevin Hughes (Edinburgh Napier University) and Arie Israel (UT at Austin).

Christoph Thiele
Hausdorff Center of Mathematics, Bonn

Title: Maximally modulated singular integrals on doubling metric measure spaces

Abstract: A maximally modulated singular integral operator has the form

$$Tf(x) = \sup_{\theta \in \Theta} \int K(x, y) e^{i\theta(y)} f(y)\, dy$$

where K is Calderón Zygmund operator and Θ is a suitable class of phase functions. Originally motivated by convergence questions of Fourier series, where K is the Hilbert kernel and Θ is the class of linear functions, more general cases have been studied including the class of polynomials of a fixed degree (Victor Lie). We generalize this further, presenting a class of axioms on Θ that allow to generalize such operators to doubling metric measure spaces. We discuss estimates and applications for these operators. We also discuss an ongoing computer verification project in Lean for some of these estimates. This project will include a computer verification of the classical theorem of Carleson on almost everywhere convergence of Fourier series. This is joint work with Lars Becker, Floris van Doorn, Asgar Jamneshan, and Rajula Srivastava.

Ciprian Demeter
Indiana University

Title: When the Schrödinger equation meets number theory

Abstract: The question of pointwise convergence of the solution of the free Schrödinger equation to the initial data has motivated significant developments in harmonic analysis. Incidence geometry, refined Strichartz estimates and decoupling are some of the tools that proved instrumental for Euclidean spatial domains. These tools seem to be ineffective in the case of tori, and the problem is open even in one spatial dimension. I will describe progress on this case that involves completely different tools, a mix of number theory and combinatorics.

David Beltran
University of Valéncia

Title: Endpoint sparse bounds

Abstract: Sparse bounds have attracted a lot of attention over the last 15 years in Harmonic Analysis. For operators beyond Calderón–Zygmund theory, it is fairly well understood that bilinear (p, q') sparse bounds are related to $L^p - L^q$ mapping properties of local versions of the operator under consideration. For this connection to be valid, the $L^p - L^q$ bounds should incorporate a slight regularity gain, and thus this approach typically yields no endpoint sparse bounds. In this talk, we will present how to bypass this lack of regularity and still obtain endpoint sparse bounds for certain classes of Fourier multipliers, including oscillatory Fourier multipliers and Bochner–Riesz multipliers. This is joint work with Joris Roos and Andreas Seeger.

Detlef Müller
CAU Kiel

Title: An FIO approach to L^p bounds for wave equations associated to sub-Laplacians

Abstract: Let $\mathcal{L}$ be a sub-Laplacian on a sub-Riemannian manifold of dimension d, for instance a left-invariant sub-Laplacian on a Carnot group. We know by now that the ranges of validity of wave propagator estimates of Miyachi–Peral type for $\mathcal{L}$ cannot be wider than the corresponding ranges for the Laplacian on $\mathbb{R}^d$.

On the other hand, until recently, the only cases of non-elliptic sub-Laplacians where it could be proved that these ranges actually do agree with the ones for the Laplacian on $\mathbb{R}^d$ have been sub-Laplacians on Heisenberg type groups, through joint works with E.M. Stein, and with A. Seeger. Both of the different approaches devised in these works had heavily made use of symmetry properties of such groups and the associated sub-Laplacians, and were very different from the usual approach to wave equations associated to elliptic Laplacian by means of FIO techniques. For many reasons, it had indeed been unclear for a long time if any FIO approach might be feasible also for non-elliptic sub-Laplacians.

In my talk I shall report on recent joint work with A. Martini, in which we have been able to devise such an approach via FIOs with complex phase for sub-Laplacians on Métivier groups, leading to the sharp L^p ranges (up to the endpoint). Such results would have been out of reach by the previously used methods. Moreover, we are confident that our approach can be extended to much wider classes of sub-Laplacians in 2-step sub-Riemannian settings.

Hong Wang
New York University

Title: Furstenberg sets estimate in the plane

Abstract: The Furstenberg set conjecture states that a set containing an s-dimensional subset of a line in every direction should have dimension at least $(3s+1)/2$ when $s > 0$. It can be viewed as an incidence problem for tubes and a continuous version of the Szemerédi-Trotter theorem.

We will survey a sequence of results by Orponen, Shmerkin and a recent joint work with Ren that lead to the solution of this conjecture.

Jim Wright
University of Edinburgh

Title: Oscillatory integrals over locally compact fields: A unified theory

Abstract: Here we consider oscillatory integrals defined over general locally compact fields $\mathbb{K}$. When $\mathbb{K} = \mathbb{R}$ is the real field, oscillatory integrals are a basic object of study in harmonic analysis. On the other hand, complete exponential sums or character sums can be realised as oscillatory integrals over the p-adic field $\mathbb{K} = \mathbb{Q}_p$. These are basic objects in number theory. In both cases, for real oscillatory integrals and exponential sums, there is an extensive literature giving sharp bounds for these oscillating entities.

In this talk, we discuss a unified theory for oscillatory integrals defined over any locally compact field. This is joint work with Gian Maria Dall'Ara.

Jonathan Bennett
University of Birmingham

Title: Adjoint Brascamp–Lieb inequalities

Abstract: The Brascamp–Lieb inequalities are generalisations of the Hölder, Loomis–Whitney and Young convolution inequalities, and have found many applications in harmonic analysis in recent years. In this talk we present an "adjoint" form of these inequalities that may be viewed as an L^p version of the entropic Brascamp–Lieb inequalities of Carlen and Cordero–Erausquin. As an application we establish lower bounds on some well-known tomographic transforms, such as the classical X-ray and Radon transforms. This is joint work with Terry Tao.

Jonathan Fraser
University of St. Andrews

Title: The Fourier spectrum and applications to exceptional sets

Abstract: Suppose X is a compact Salem set in the plane with Hausdorff dimension less than 1. It is easy to see that there are no exceptional directions in Marstrand's theorem: the Hausdorff dimension is preserved under all orthogonal projections. The question I will consider is whether we can say more than this, that is, when can Fourier decay be used to improve known bounds for the Hausdorff dimension of the exceptional set? I will formulate partial progress towards this question in terms of the Fourier spectrum, which is a continuously parametrised family of dimensions living between the Fourier and Hausdorff dimension. This is joint work with Ana de Orellana (St Andrews).

Jonathan Hickman
University of Edinburgh

Title: Variable coefficient L^p local smoothing

Abstract: I will discuss some recent work concerning variable coefficient extensions of L^p local smoothing estimates for the Schrödinger propagator. This can be thought of as a counterpart to a classical oscillatory integral operator bound of Bourgain (1991). Whilst Bourgain's result relies on studying Kakeya sets of curves, our L^p local smoothing result relies on studying Nikodym sets of curves. An important observation of Wisewell (2005) is that the Nikodym theory is surprisingly different from the Kakeya theory. Our work aims to further investigate and exploit these differences.

Joris Roos
University of Massachusetts–Lowell

Title: Isoperimetric and Poincaré inequalities on the Hamming cube

Abstract: The talk will be about isoperimetric inequalities on the Hamming cube near and at the critical exponent $1/2$. These are closely related to L^1 Poincaré inequalities for functions on the Hamming cube and corresponding inequalities in Gaussian space. In joint work with Polona Durcik and Paata Ivanisvili, we improve currently known bounds by proposing a new Bellman-style function satisfying a certain two-point inequality. In particular, we obtain a sharp estimate near the critical exponent. Once a good candidate function is found, the key difficulty remains verification of the two-point inequality where we benefit from computer-assisted proofs.

Joshua Zahl
University of British Columbia

Title: Curve tangencies and maximal functions

Abstract: I will discuss a class of maximal operators that arise from averaging functions over thin neighborhoods of curves in the plane. Examples of such operators are the Kakeya maximal function and the Wolff and Bourgain circular maximal functions. To understand the behavior of these operators, we need to study the possible intersection patterns for collections of curves in the plane: how often can these curves intersect, how often can they be tangent, and how often can they be tangent to higher order?

Krystal Taylor
The Ohio State University

Title: How to use Projections to Cover a Fractal Set by Curves without wasting measure

Abstract: A classic theorem of Davies states that a set of positive Lebesgue measure can be covered by lines in such a way that the union of the set of lines has the same measure as the original set. This surprising and counter-intuitive result has a dual formulation in the form of a prescribed projection theorem. We investigate an analogue of these results in which lines are replaced by shifts of a fixed curve.

In particular, we show that a measurable set in the plane can be covered by translations of a fixed open curve, obeying some mild curvature assumptions, in such a way that the union of the translated curves has the same measure as the original set. Our results rely on a Venetian blind construction and extend to transversal families of projections. As an application, we consider how duality between curves and points can be used to construct nonlinear Kakeya sets.

Larry Guth
Massachusetts Institute of Technology

Title: A new approach to bounding large values of Dirichlet polynomials

Abstract: Bounds for Dirichlet polynomials help to bound the number of zeroes of the Riemann zeta function in vertical strips, which is relevant to the distribution of primes in short intervals. A Dirichlet polynomial is a trigonometric polynomial of the form

$$D(t) = \sum_{n=N}^{2N} b_n n^{it}.$$

The main question is about the size of the superlevel sets of $D(t)$. We normalize so that the coefficients have norm at most 1, and then we study the superlevel set $\{|D(t)| > N^\sigma\}$ for some exponent σ between 1/2 and 1.

For large values of σ, Montgomery proved very strong bounds for the superlevel sets. But for $\sigma \leq 3/4$, the best known bounds follow from a very simple orthogonality argument (and they don't appear to be sharp). We improve the known bounds for σ slightly less than 3/4. This is work in progress. Joint with James Maynard.

Malabika Pramanik
University of British Columbia

Title: Numbers – are they normal?

Abstract: They say the only normal people are the ones you don't know very well. What about numbers? Which ones are normal, and how well do we know them?

The notion of mathematical normality is related to the occurrence of different digits in a number. Roughly speaking, a normal number is one in which every block of digits appears with the same limiting frequency. For example, 0.12345678910111213... is normal in base 10, but 0.1212121212... is not. Normality of numbers is connected to many areas of mathematics, like diophantine approximation, ergodic theory, geometric measure theory, analysis and computer science.

We will discuss a few open problems about normal numbers that lie at the intersection of harmonic analysis and measure theory, and mention some recent progress on them.

Philip Gressman
University of Pennsylvania

Title: Generalized sublevel set inequalities for differential forms

Abstract: This talk will discuss the problem of uniformly estimating certain geometric integrals which appear in a recently-developed testing condition for the L^p-improving properties of multilinear Radon-like transforms. The problem shares features of a classical sublevel set inequality but involves differential forms rather than scalar-valued functions. This additional geometric structure turns out to play an important role and leads to results which are indeed powerful enough to make the testing condition a computationally-viable tool in many cases. As an application, we will discuss a new characterization of the family of Radon-like transforms known as model operators which is phrased in terms of the notion of semistability from geometric invariant theory.

Polona Durcik
Chapman University

Title: Quantitative norm convergence of triple ergodic averages for commuting transformations

Abstract: We discuss a quantitative result on norm convergence of triple ergodic averages with respect to three general commuting transformations. For these averages we prove an r-variation estimate, $r > 4$, in the norm. We approach the problem via real harmonic analysis, using the recently developed techniques for bounding singular Brascamp-Lieb forms. It remains an open problem whether such norm-variation estimates hold for all $r \geq 2$ as in the cases of one or two commuting transformations, or whether such estimates hold for any $r < \infty$ for more than three commuting transformations. This is joint work with Lenka Slavíková and Christoph Thiele.

Rodrigo Bañuelos
Purdue University

Norm estimates for discrete singular integrals

Abstract: We discuss sharp norm estimates for discrete singular integrals on the d-dimensional lattice, $d \geq 2$. The operators are constructed as conditional expectations of martingale transforms. The results are motivated by the classical case of the Hilbert transform on the integers.

The talk is based on work with Mateusz Kwaśnicki (Wrocław University, Poland) and Daesung Kim (Georgia Tech).

Ruixiang Zhang
UC Berkeley

Title: A new conjecture to unify Fourier restriction and Bochner-Riesz

Abstract: Building on prior research regarding a question posed by Hörmander, we recently formulated a new conjecture to unify Fourier restriction and Bochner-Riesz and found some evidence. I will talk about the history and mention some interesting components in the proofs. Joint work with Shaoming Guo and Hong Wang.

Tuomas Hytönen
Aalto University

Title: Convex body domination and other reduced inequalities for vector-valued functions

Abstract: Over the past twelve years or so, sparse domination has become a new standard for capturing information about the singularities of various operators. In the setting of vector-valued functions, however, the basic form of this method loses critical directional information that is relevant, for instance, for matrix-weighted estimates. To address this shortcoming, Nazarov, Petermichl, Treil, and Volberg introduced a refined notion called convex body domination. Subsequently, several sparse domination results have been improved to convex body domination by various authors. In this talk, I present a general framework for such results and some recent applications, including a refined Kato-Ponce inequality (or fractional Leibniz rule) for vector-valued functions.

Xiumin Du
Northwestern University

Title: Weighted refined decoupling and Falconer distance set problem

Abstract: In this talk, I'll discuss a refinement of Bourgain–Demeter's decoupling inequality in the weighted setting and the case that the directions of wave packets are in a small neighborhood of a subspace. Such inequalities arise from the study of Falconer distance set problem. Combining weighted refined decoupling and new radial projection estimates by Ren, we proved the following result: if a compact set $E \subset \mathbb{R}^d$ has Hausdorff dimension larger than $\frac{d}{2} + \frac{1}{4} - \frac{1}{8d+4}$, where $d \geq 4$, then there is a point $x \in E$ such that the pinned distance set $\Delta_x(E)$ has positive Lebesgue measure. The result also holds for dimension $d = 3$, but it requires more geometric input (Yumeng will say more about this in her talk). Joint work with Yumeng Ou, Kevin Ren, and Ruixiang Zhang.

Yumeng Ou
University of Pennsylvania

Title: New improvement to Falconer's distance set problem

Abstract: Falconer's distance set conjecture says that if a compact set E in $\mathbb{R}^d$ has Hausdorff dimension greater than $\frac{d}{2}$, then its distance set $\Delta(E)$ must have positive Lebesgue measure. In this talk, I'll discuss a recent improvement that we obtained for the problem in dimension three and higher, which says that it's sufficient to have dimension greater than $\frac{d}{2} + \frac{1}{4} - \frac{1}{8d+4}$. The proof relies on a recent radial projection theorem of Ren in a crucial way. This is joint work with Xiumin Du, Kevin Ren, and Ruixiang Zhang. (Also see Xiumin's talk for a different proof of our result.)