Matthew Blair

Gaussian beam approximations on Riemannian manifolds and applications

Abstract: We consider high-frequency Gaussian beam approximations to the wave equation on a Riemannian manifold. These are approximate solutions whose phase space profile is highly concentrated along a geodesic on scales which saturate the uncertainty principle. In particular, we are interested in constructing approximations that are well-behaved under coordinate transformations and seek to analyze their dynamics over large, frequency-dependent time intervals. We then show how such constructions can be applied towards the analysis of eigenfunctions of the Laplacian on a compact Riemannian manifold. Of particular interest here are estimates on the microlocal Kakeya-Nikodym norms introduced in previous works with C. Sogge, which have implications for L^p bounds on eigenfunctions.

Raluca Felea

FIOs with singularities

Abstract: I will talk about the results obtained in collaboration with Allan Greenleaf regarding the composition calculus of FIOs with singularities like folds, blowdowns, cusps and submersions with folds. We focus on the composition operator F^*F, and show that this operator has a kernel which belongs to a class of distributions associated to cleanly intersecting lagrangians, in some cases, or a kernel which belongs to a class of distributions associated to a singular lagrangian, in other cases.

Michael Greenblatt

Hessian Determinants and Averaging Operators over Surfaces in $\mathbb{R}^3$

Abstract: We describe $L^p(\mathbb{R}^3)$ to $L^p_s(\mathbb{R}^3)$ Sobolev improvement theorems for local averaging operators over real analytic surfaces in $\mathbb{R}^3$. For most such operators, in a sense made precise, the set of (p, s) for which we have $L^p(\mathbb{R}^3)$ to $L^p_s(\mathbb{R}^3)$ boundedness is optimal up to endpoints. Using an interpolation argument in conjunction with these $L^p(\mathbb{R}^3)$ to $L^p_s(\mathbb{R}^3)$ results we also have an $L^p(\mathbb{R}^3)$ to $L^q(\mathbb{R}^3)$ improvement theorem, and the set of exponents (p, q) obtained will also usually be optimal up to endpoints.

Philip Gressman

The Selective Summation Inequalities of
Nonconcentration and Superorthogonality

Abstract: We will discuss several families of fundamental inequalities that have the property that they bound structured sums of terms by sums of strict subsets of those terms (a feat which would be plainly impossible to accomplish for arbitrary sums). Key applications/examples of such inequalities appear when, for example, making a priori transversality assumptions in proofs of restriction or decoupling. In particular, we will discuss a new nonpositive family of inequalities of this sort which have interesting and unifying consequences for the notion of "superorthogonality".

Eric Grinberg

A Backward Extrapolation of Determinants, to Laplace and Leibniz and beyond, with Parallels in Integral Geometry

Abstract: Determinants are ubiquitous across the areas of mathematics, and Integral Geometry is no exception. We remark on the role of determinants in Radon transforms and their inversions, and discuss the core notion of determinant, how it was and is introduced, and how it might be; its geometric and perhaps non-geometric aspects. The Cavalieri principle and conditions figure in the discussion.

Matti Lassas

Inverse problems for non-linear partial differential equations:
How non-linearity makes imaging easier?

Abstract: In the talk we give an overview on how inverse problems can be used solved using non-linear interaction of the solutions. This method can be used for several different inverse problems for non-linear hyperbolic or elliptic equations. In this approach one does not consider the non-linearity as a troublesome perturbation term, but as an effect that aids in solving the problem. Using it, one can solve inverse problems for non-linear equations for which the corresponding problem for linear equations is still unsolved.

As an example, we consider the non-linear wave equation $\Box_g u + u^m = f$ on a Lorentzian manifold $M \times R$ and the source-to-solution map $\Lambda_V : f \to u|_V$ that maps a source f, supported in an open domain $V \subset M \times R$, to the restriction of u in V. Under suitable conditions, we show that the observations in V, that is, the map Λ_V, determine the metric g in a larger domain which is the maximal domain where signals sent from V can propagate and return back to V. The proofs used to study these inverse problems are based on microlocal analysis, in particular on the propagation of singularities and the products of conormal distributions studied by A. Greenleaf and G. Uhlmann. These tools are combined with Lorentzian geometry to determine the structure of the spacetime.

Adrian Nachman

A Nonlinear Plancherel Theorem with Applications to Global Well-posedness for the Defocusing Davey-Stewartson Equation and to the Calderón Inverse Problem in Dimension two

Abstract: We consider a well-studied nonlinear Fourier transform in two dimensions for which a proof of the Plancherel theorem had been a challenging open problem. The talk will explain the main ideas involved in the solution of this problem, as well as in the solution of two other open problems that motivated it: global well-posedness for the defocusing DSII equation in the mass critical case, and global uniqueness for the inverse boundary value problem of Calderon for a class of unbounded conductivities. Included will be two theorems of independent interest: new estimates for classical fractional integrals, and a new result on L^2 boundedness of pseudodifferential operators with non-smooth symbols. (All of this is joint work with Idan Regev and Daniel Tataru.)

Clifford Nolan

Applications of Microlocal Analysis to Imaging in Seismology and Radar

Abstract: In this talk, I will review a range applications of microlocal analysis to imaging in the fields of seismology and radar. In fact, the range of applications is broader than just these two fields and I will mention why this is so during the presentation.

Imaging in this context relies upon inferring singularities in material properties (e.g., density, electrical permittivity, etc) from the singular component of waves (e.g, ultrasound or radio waves), which have scattered internally within the material to be imaged. These scattered waves are generated and recorded using energy sources and receivers in various configurations on the boundary of the material. Singularities in this context means the wavefront set of a distribution.

The scattering process is normally modelled using a scattering operator, which often turns out to be a Fourier Integral Operator. The associated wavefront relation of the Fourier Integral Operator is a Lagrangian submanifold, the geometry of which is crucial to understand if one is to have any hope of obtaining a reliable estimate of the singularities in the material properties and to present this estimate in the form of an image of the material properties.

Eyvindur Palsson

Point Configurations and Geometric Averaging Operators

Abstract: Two classic questions - the Erdős distinct distance problem, which asks about the least number of distinct distances determined by points in the plane, and its continuous analog, the Falconer distance problem - both focus on distance. Here, distance can be thought of as a simple two point configuration. When studying the Falconer distance problem, a geometric averaging operator, namely the spherical averaging operator, arises naturally. Questions similar to the Erdős distinct distance problem and the Falconer distance problem can also be posed for more complicated patterns such as triangles, which can be viewed as 3-point configurations. In this talk I will go through some of the history of such point configuration questions for triangles and end with recent progress on Mattila-Sjölin type theorems for triangles.

Christopher Sogge

Product Manifolds with Improved Spectral Cluster and Weyl Remainder Estimates

Abstract: We describe joint work with Xiaoqi Huang and Michael Taylor. We explore the problem of when one can obtain improved spectral and Weyl remainder estimates, especially on product manifolds. In particular, for products of spheres of length 5 or more we obtain optimal L^q estimates for eigenfunctions for q sufficiently large in a result that generalizes a classical result of Walfisz for the torus. This result, although for a special class of manifolds and involving large exponents, is a natural analog of the Stein-Tomas restriction/extension theorem for Euclidean space. It and other of our results are also motivated by recent work of Iosevich and Wyman.

Betsy Stovall

On extremizing sequences for adjoint Fourier restriction to the sphere

Abstract: In this talk, we will describe recent work with Taryn C. Flock developing a linear profile decomposition for the $L^p \to L^q$ adjoint Fourier restriction operator associated to the sphere, valid for exponent pairs $p < q$ for which this operator is bounded. This result, new when $p \neq 2$, has implications for the behavior of extremizing sequences for the spherical extension operator, including new existence results for extremizers for certain values of p, q. In particular, the $L^p \to L^q$ extension operator is extremizable more often than not.

Brian Street

Maximal Subellipticity

Abstract: The theory of elliptic PDEs stands apart from many other areas of PDEs because sharp results are known for very general linear and fully nonlinear elliptic PDEs. Many of the classical techniques from harmonic analysis were first developed to prove these sharp results; and the study of elliptic PDEs leans heavily on the Fourier transform and Riemannian geometry.

Starting with work of Hörmander, Kohn, Folland, Stein, and Rothschild in the 60s and 70s, a far-reaching generalization of ellipticity was introduced: now known as maximal subellipticity or maximal hypoellipticity. In the intervening years, many authors have adapted results from elliptic PDEs to various special cases of maximally subelliptic PDEs.

Where elliptic operators are connected to Riemannian geometry, maximally subelliptic operators are connected to sub-Riemannian geometry. The Fourier transform is no longer a central tool but can be replaced with more modern tools from harmonic analysis.

In this talk, we present the sharp regularity theory of general linear and fully nonlinear maximally subelliptic PDEs.

Gunther Uhlmann
40 Years of Calderón's Problem