Mathematics 623 – Complex Analysis
Fall 2014

Assignment 9
Due Wednesday, November 12

A. Exercises no 19, 20 on pages 106/107 and exercise 1 and 9 on pages 248/249.

B1. (i) Let $S = \{z : 0 \le \mathrm{Re}\ (z) \le 1\}$. Let f be continuous in S and analytic in the interior of S and suppose that

$$|f(iy)| \le 1 \text{ for all } y \in \mathbb{R}$$
$$|f(1+iy)| \le 1 \text{ for all } y \in \mathbb{R}$$
$$|f(x+iy)| \le Ce^{|y|} \text{ for all } z = x+iy \in S.$$

Show that
$$|f(z)| \le 1 \text{ for all } z \in S.$$

(ii) Let S be as in (i) and let f be continuous in S and analytic in the interior of S. Suppose that

$$|f(iy)| \le M_0 \text{ for all } y \in \mathbb{R}$$
$$|f(1+iy)| \le M_1 \text{ for all } y \in \mathbb{R}$$
$$|f(x+iy)| \le Ce^{|y|} \text{ for all } z = x+iy \in S.$$

Show that
$$|f(x+iy)| \le M_0^{1-x} M_1^x \text{ for } 0 < x < 1,\ y \in \mathbb{R}.$$

Hint: For large R, apply the maximum principle to $f(z)e^{\varepsilon z^2}$ in the rectangle with corners at iR, $-iR$, $1+iR$, $1-iR$. For part (ii) consider $f(z)M_0^{-(1-z)}M_1^{-z}$.

B2. Compute
$$\oint_\Gamma \frac{z^3}{(z-1)^2(z+2)(z-i)}dz$$
where Γ is the circle of radius 4, centered at 0, with the counterclockwise orientation.