Mathematics 623 – Complex Analysis
Fall 2014

Assignment 7.
Due Wednesday, October 29

1. Let f and g be entire functions and suppose that $|f(z)| \le |g(z)|$ for all $z \in \mathbb{C}$.
Prove that there exists a complex number a with $|a| \le 1$ so that $f(z) = ag(z)$ for all $z \in \mathbb{C}$.

2. Problem No. 12 on p. 105 in the textbook.

3. Problem No. 13 on p. 105 in the textbook.

4. Let $f(z) = \sum_{n=0}^{\infty} a_n z^n$ where the power series converges for all $|z| < 1$. Let $|w| = 1$ and suppose that there exists an $R > 1$ so that f extends to a holomorphic function in $\{z : |z| < R, z \ne w\}$ and suppose that f has a pole at w.
Show that
$$\lim_{n\to\infty} \frac{a_n}{a_{n+1}} = w\,.$$

Hint. Let $\gamma_\epsilon(w)$ be the circle of radius ε centered at w and let Γ_δ be the circle of radius $1+\delta$ centered at 0. For small δ and $\epsilon < \delta$ consider the integrals
$$\int_{\gamma_\epsilon} \frac{f(z)}{z^{n+1}} dz, \qquad \int_{\Gamma_\delta} \frac{f(z)}{z^{n+1}} dz$$
and relate them to the coefficient a_n.