Mathematics 623 – Complex Analysis
Fall 2012
Assignment 4

1. Let $\partial B_r(z_0)$ the boundary of the disc of radius r centered at z_0, with the counterclockwise orientation. Compute the integrals
(i)

$$\int_{\partial B_2(0)} \frac{e^z}{(z+1)(z-3)^2}\, dz$$

(ii)

$$\int_{\partial B_2(0)} \frac{\sin z}{z+i}\, dz$$

(iii)

$$\int_{\partial B_2(-2i)} \frac{1}{z^2+1}\, dz$$

(iv)

$$\int_{\partial B_1(0)} \frac{e^z}{(z-2)^3}\, dz$$

2. Let $\rho > 0$ and assume that f is continuous on the closed ball $\overline{B}_\rho(0)$ and holomorphic in the open ball $B_\rho(0)$. Let $w \in B_\rho(0)$. Prove

$$f(w) = \frac{1}{2\pi i} \int_{\partial B_\rho(0)} \frac{f(z)}{z-w}\, dz\,.$$

3. Prove

$$\int_0^\infty \frac{\sin x}{x} dx = \frac{\pi}{2}\,.$$

4. Let $0 < a < 1$. Evaluate

$$\int_0^{2\pi} \frac{d\theta}{1 - 2a\cos\theta + a^2}\,.$$