Mathematics 623 – Complex Analysis
Fall 2014

Assignment 10
Due Wednesday, December 3

A. Exercises 4, 5, 10, 11, 12, 14 in §8.5. of the textbook.

B1.

(i) Let T be a fractional linear transformation such that $T(0) = 0$, $T(1) = 1$ and $T(\infty) = \infty$. Prove that $T(z) = z$ for all $z \in \mathbb{C}$.

(ii) Let z_1, z_2, z_3 be distinct complex numbers and let $F(z) = \frac{z-z_1}{z-z_2}\frac{z_3-z_2}{z_3-z_1}$. Prove that F is the *unique* fractional linear transformation with $F(z_1) = 0$, $F(z_2) = \infty$, $F(z_3) = 1$.

(iii) Let z_1, z_2, z_3 be distinct complex numbers and let w_1, w_2, w_3 be distinct complex numbers. Prove that there is a unique fractional linear transformation G with $G(z_1) = w_1$, $G(z_2) = w_2$, $G(z_3) = w_3$.

B2.

(i) Find a biholomorphic map from $\Omega_1 = \{z \in \mathbb{C} : \mathrm{Re}(z) < 0, \mathrm{Im}(z) < 0\}$ and $\Omega_2 = \{z \in \mathbb{C} : z = x + iy, 4 < x < 6, -\infty < y < \infty\}$.

(ii) Find a biholomorphic map from $\Omega_3 = \{z = x + iy : x + y < -3\}$ to $\Omega_4 = \{z : |z - 2| < 5\}$.

(iii) Find a biholomorphic map from $\Omega_5 = \{z : |z| < 1\}$ to $\Omega_6 = \{z : \mathrm{Im}(z) < 0\}$, so that $f(i/2) = -2i$.