Mathematics 623 – Complex Analysis
Fall 2014
Due September 12
First assignment

1. Find the real and imaginary parts, and the absolute values of
(i) $(i+1)(i-2)(i+3)$, (ii) $\left(\frac{2+3i}{2-3i}\right)^2$, (iii) $\frac{1}{x+iy}$.

2. Determine all complex numbers z satisfying the equation $z^4 - 2iz^2 - 2 = 0$.

3. Exercise 1 on p. 24.

4. Find all solutions of the equation $z^5 = -1024i$.

5. Exercise 7 on p. 26/27.

6. Exercise No 4. on p. 26/27.

7. Let n, k be positive integers and $\omega = \cos\frac{2\pi}{n} + i\sin\frac{2\pi}{n}$. Determine the value of

$$1 - \omega^k + \omega^{2k} - \cdots + (-1)^{n-1}\omega^{(n-1)k}.$$

8.* Prove for complex numbers z_i, w_i

$$\sum_{i=1}^{n} |z_i|^2 \sum_{i=1}^{n} |w_i|^2 - \Big|\sum_{i=1}^{n} z_i w_i\Big|^2 = \sum_{1\le i<j\le n} |z_i\overline{w}_j - z_j\overline{w}_i|^2$$