Analysis I-II-III (Math 521-522-621)

An introductory sequence in analysis

Analysis I - Math 521

Prerequisites for Math 521: Previous serious exposure to understanding and writing proofs is required in Math 521. Any of the courses Math 341, Math 371, Math 375-76, Math 421, or Math 461 provide this experience. Admission to Math 521 is also possible with the consent of the instructor.

Course Outline

1. The real number system.

An axiomatic approach to the real numbers $\mathbb{R}$; the explicit construction is not part of this course.

2. Metric spaces and basic topology.

Finite, countable and uncountable sets. Metric spaces, compact sets, connected sets, perfect sets.

3. Sequences and series.

Convergence, Cauchy sequences, monotone sequences, upper and lower limits, general properties of series, series with nonnegative terms, the role of the geometric series, the number e, summation by parts, absolute and conditional convergence, multiplication of series, rearrangements.

Uniform convergence of a sequence (series) of functions.

4. Continuity.

Limits of functions, continuous functions, continuity and compactness, continuity and connectedness.

5. Topics from differential and integral calculus.

Review of the basics, with some proofs (a repetition of math 421 is *not* intended here). Uniform continuity and the existence of the integral for continuous functions. More on the Riemann integral. Fundamental theorem of calculus and Taylor's theorem.

Uniform convergence and integration, uniform convergence and differentiation. Power series.

Improper integrals.

Note: The order of topics is flexible. It will naturally depend on the choice of the textbook and the preferences of the instructor. For example, one can study sequences and series in the beginning, and introduce metric spaces later. It is important to cover uniform convergence in Math 521.

Analysis II - Math 522

Prerequisites for Math 522: Math 521, a course in linear algebra (equivalent of Math 340 or 341 or 375) which can also be taken concurrently, or consent of instructor.

Course Outline

We follow here the outline in Rudin's book. The order of the topics in Math 522 is quite flexible.

Review of some topics that may not have been covered in Math 521.

6. More on convergence.

Approximations of the identity. Approximation by polynomials, the Stone-Weierstrass theorem. Infinite products (optional).

7. Special functions.

Exponential functions, and more on power series. Algebraic completeness of the complex field. Fourier series. Stirling's formula and the Γ-function.

8. Compactness in metric spaces.

Characterizations of compactness in metric spaces, The Arzela-Ascoli theorem (with a concrete application such as the Peano's existence theorem for differential equations).

9. The contraction principle.

With applications, in particular existence and uniqueness theorems for differential equations.

10. Differential calculus in normed spaces.

Including the implicit function theorem and applications.

11. Other optional topics.

Such as:
Rectifiability of curves.
The construction of real numbers.
Baire category with some applications.

Analysis III - Math 621

To be offered in the Spring of 2011.

Prerequisites for Math 621. Math 521-522, or consent of instructor.

Course Outline

Several approaches to the subject are possible. The following is based on Spivak's text "Calculus on manifolds", but topics could certainly be covered in different order, and with different emphasis.

1. Differentiation. Short review of differential calculus in several variables.

2. Integration in Euclidean spaces. Basic definitions, measure and content zero, and characterization of Riemann integrability (optional). Fubini's theorem. Partition of unity. Changes of variables.

3. Some multilinear algebra. Review of determinants. Multilinear maps, tensors, alternating tensors, wedge product.

4. Fields and forms. Vector fields, differential forms, differential of a form, closed and exact forms.

5. Manifolds. Basic concept of a manifold, definition(s) of tangent space, fields and forms on manifolds, orientation.

6. Integration. Stokes' theorem on manifolds.

Euclidean measure for submanifolds of $\mathbb{R}^n$ and the classical theorems in vector analysis by Green, Gauss and Stokes. The form $f(z)dz$ and Cauchy's integral theorem.

7. Other optional topics (if time permits).