Mathematics 522
Homework assignment No.2.

Due Friday, September 27.

1. For complex numbers z define

$$\sinh z = \sum_{k=0}^{\infty} \frac{z^{2k+1}}{(2k+1)!}$$

$$\cosh z = \sum_{\ell=0}^{\infty} \frac{z^{2\ell}}{(2\ell)!}$$

(i) Show that these series converge for all z and thus the functions sinh and cosh are well defined.

(ii) what is the relation between exp, cosh, sinh?

If we define $\cos z = \cosh(iz)$, $\sin z = \frac{1}{i}\sinh(iz)$ what is the relation between exp, cos, sin?

(iii) Show that

$$\sinh(z+w) = \sinh z \cosh w + \cosh z \sinh w$$

$$\cosh(z+w) = \cosh z \cosh w + \sinh z \sinh w$$

Derive similar formulas for $\sin(z+w)$, $\cos(z+w)$.

2.-3.-4.-5. Problems 4,5,6 and 9 on p.165-166 in Rudin's book.

6. Suppose the real-valued functions f_n and g_n are defined on an interval E (or more generally on a set E).

Suppose that

(a) the partial sums of the series $\sum f_n(x)$ are uniformly bounded on E,

(b) $g_n \to 0$ uniformly on E

(c) $g_k(x) \geq g_{k+1}(x)$ for all $k = 1, 2, \ldots$ and all $x \in E$.

Prove that $\sum f_n g_n$ converges uniformly on E

7. Prove that $\sum_{k=0}^{\infty}(-1)^k \frac{x^{2k+1}}{2k+1}$ converges uniformly on $[-1,1]$ and evaluate the sum. What do you get for $x = 1$?

8. Evaluate $\sum_{k=1}^{\infty} k^2 x^{2k+1}$ for $|x| < 1$.