Math 521-522

An introductory sequence in analysis

Textbook: There are numerous good choices, for example, W. Rudin, *Principles of Mathematical Analysis*, S. Lang *Undergraduate Analysis*, R. Strichartz, *The Way of Analysis.*

Math 521

Course Outline

1. The real number system.

An axiomatic approach to the real numbers $\mathbb{R}$; the explicit construction is not part of this course.

2. Metric spaces and basic topology.

Finite, countable and uncountable sets. Metric spaces, compact sets, connected sets, perfect sets.

3. Sequences and series.

Convergence, Cauchy sequences, monotone sequences, upper and lower limits, general properties of series, series with nonnegative terms, the role of the geometric series, the number e, summation by parts, absolute and conditional convergence, multiplication of series, rearrangements.

Uniform convergence of a sequence (series) of functions.

4. Continuity.

Limits of functions, continuous functions, continuity and compactness, continuity and connectedness.

5. Topics from differential and integral calculus.

Review of the basics, with some proofs (a repetition of math 421 is *not* intended here). Uniform continuity and the existence of the integral for continuous functions. More on the Riemann integral. Fundamental theorem of calculus and Taylor's theorem.

Uniform convergence and integration, uniform convergence and differentiation. Power series.

Improper integrals (could be postponed to Math 522).

Note: The order of topics is flexible. It will naturally depend on the choice of the textbook and the preferences of the instructor. For example, one can study sequences and series in the beginning, and introduce metric spaces later. It is important to cover uniform convergence in Math 521.

Math 522

Course Outline

Review of some topics that may not have been covered in Math 521.

6. More on convergence.
Approximations of the identity. Approximation by polynomials, the Stone-Weierstrass theorem. Infinite products (optional).

7. Special functions.
Exponential functions, and more on power series. Algebraic completeness of the complex field. Fourier series. Stirling's formula and the Γ-function.

8. The contraction principle.
With applications, in particular existence and uniqueness theorems for differential equations.

9. Differential calculus in normed spaces.
Including the implicit function theorem and applications.

10. Compactness in metric spaces.
Characterizations of compactness in metric spaces, The Arzela-Ascoli theorem (with a concrete application such as the Peano's existence theorem for differential equations).

11. Other optional topics.
Rectifiability of curves.
The construction of real numbers.
Baire category with some applications.

The order of the topics in Math 522 is flexible.