Math 222 – Review problems for Dec. 14.

1. Let L be the line given by the parametric equation

$$\mathbf{x}(t) = \begin{pmatrix} 1-2t \\ 1-3t \\ 1-t \end{pmatrix}$$

Let $\mathcal{P}$ be the plane which passes through the point $(1,0,0)$ and which is perpendicular to L.

(i) Find the equation for the plane $\mathcal{P}$.

(ii) Find two vectors of magnitude 5 which are parallel to L.

(iii) Find two vectors of magnitude 3 which are both parallel to the plane $\mathcal{P}$ and perpendicular to $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$.

(iv) Find the distance of the origin to $\mathcal{P}$.

(v) Find the distance of the origin to the line L.

(vi) Find the distance of the point $Q(4,1,2)$ to the plane $\mathcal{P}$ and to the line L.

(vii) Let A be the point where $\mathcal{P}$ and L intersect, let B be the point in $\mathcal{P}$ which lies on the x_1-axis and let C be the point in $\mathcal{P}$ which lies on the x_3-axis.

Compute the area of the triangle ABC.

2. Compute the following integrals:

(i)

$$\int_0^\pi x(\sin x)^2 dx.$$

Hint: It helps to use a trigonometric formula for $\cos 2x$

(ii)

$$\int_0^{1/2} e^{\arcsin t} \frac{1}{(1-t^2)^{1/2}} dt.$$

(iii)

$$\int_0^a \ln(1-x^2) dx$$

for $|a| < 1$.

Hint: First simplify $\ln(1-x^2)$.

3. (i) Sketch the graph of the curve in the plane given by

$$\mathbf{x}(t) = \begin{pmatrix} e^t \sin t \\ e^t \cos t \end{pmatrix}, \qquad -\pi \le t \le \pi$$

and find its length.

(ii) Check that the point $(0,1)$ is on the curve and find an equation for the tangent line at that point.

4. Let
$$f(x) = 5\cos(x^2).$$
(i) Find a polynomial p with the property that $|f(x) - p(x)| \leq 10^{-4}$ for $-10^{-1} \leq x \leq 10^{-1}$. Justify your choice.

(ii) Show that one can find a polynomial $q(x)$ so that
$$\lim_{x \to 0} \frac{f(x) - q(x) - e^x}{x^4}$$
exists (as a finite number) and is different from 0. What is the value of this limit?

5. Solve the following initial value problems (for t near the initial time).

(i)
$$y'(t) = \frac{f(t)(y^2(t) - 3)}{4y(t)}, \qquad y(5) = -2.$$

(ii)
$$y'(t) = \frac{f(t)(y^2(t) - 3)}{4y(t)}, \qquad y(5) = 2.$$

(iii)
$$y'(t) = \frac{f(t)(y^2(t) - 3)}{4y(t)}, \qquad y(5) = -1/2.$$

(a) for a general but given continuous function f, (b) for $f(t) = t^2$.

6. Solve, for t near 2,
$$y'(t) = \frac{e^{t^2}(y(t)^2 + 1)}{t}, \qquad y(2) = -1.$$