Math 222 – Review problems on differential equations.

1. (i) Find the solution of the initial value problem $y'(x) = y(x)^2 \cos(x/2)$ with $y(\frac{\pi}{2}) = 2$.

(ii) Find the solution of the initial value problem $y'(x) = y(x)^2 \cos(x/2)$ with $y(\frac{\pi}{2}) = 0$.

2. Solve the initial value problem $y'(t)y(t) = \cos^2 t$, y(0) = 0.

- **3.** Solve the initial value problem $y''(x) \sec x = 1$, y(0) = 5, y'(0) = -2.
- 4. Solve the initial value problem y'(x)y(x) = x, y(2) = -5.

5. For every number A solve the initial value problem $y'(x)\cos^2 x = y(x)^3$, y(0) = A. You may have to distinguish between several cases.

- 6. (i) Find all functions g satisfying q''(t) 15q'(t) + 54q(t) = 0.
- (ii) Find all functions g satisfying $g''(t) 15g'(t) + 54g(t) = t^2$.
- (iii) Find all functions u satisfying $u''(x) 15u'(x) + 54u(x) = e^{-3x}$.
- (iv) Find all functions g satisfying $g''(t) 15g'(t) + 54g(t) = e^{-3t}$ and g(0) = 1.
- (v) Find all u satisfying u''(x) 15u'(x) + 54u(x) = 0, u(0) = 0, and u'(0) = 0.
- (vi) Find all v satisfying $v''(x) 15v'(x) + 54v(x) = e^{-x}$, v(0) = 0, and v'(0) = 0.
- 7. (i) Solve the initial value problem $y'(x) + \frac{x}{1+x^2}y(x) = 0$, y(0) = A. (ii) Solve the initial value problem $y''(x) + \frac{x}{1+x^2}y'(x) = 0$, y(0) = A, y'(0) = B.

8. Solve the initial value problem

$$y'(x)y(x) + x^2y(x)^3 = 0,$$
 $y(2) = 3.$

9. Solve the following initial value problems (use definite integrals in most of the answers!)

(i) y'(x) + xy(x) = 0, y(0) = A.(ii) y'(x) + xy(x) = 0, y(1) = A.(iii) $y'(x) + xy(x) = \cos x$, y(0) = A.(iv) $y'(x) + xy(x) = \cos x$, y(2) = A.y(2) = A, for any function f which is continuous on an (v) y'(x) + xy(x) = f(x),

interval containing 2. (vi) $y'(x) + xy(x) = e^{-x^2/2} \cos x$,

y(1) = A.

(vii) $y''(x) + xy'(x) = e^{-x^2/2} \cos x$, $y(1) = A, \quad y'(1) = B.$

10. You are given the information that $Y(x) = (\cos x)^{-1}$ is a solution of the differential equation $y''(x) - y(x) = \frac{2}{(\cos x)^3} - \frac{2}{\cos x}$, on the interval $(-\pi/2, \pi/2)$.

(i) Prove this.

(ii) Find the general solution of the equation.

(iii) Find the solution y that satisfies y(0) = 2 and y'(0) = 0.

11. Solve the initial value problem y''(x) + 5y'(x) + 6y(x) = f(x), y(0) = a, y'(0) = bfor the choices

(i) f(x) = 0, (ii) $f(x) = 3\cos x$, (iii) $f(x) = 6e^{-2x} + e^x$, (iv) $f(x) = 7e^{-x}\cos 2x$, (v) $f(x) = x^2 + 1.$

Same question for the problem y''(x) + 2y'(x) + 5y(x) = f(x), y(0) = a, y'(0) = b.

(Note: Your instructor has not checked whether any of the answers work out with "nice" numbers. Set up the correct linear systems for the "constants".)

12. Suppose that for a given f, the function Z satisfies Z''(x) + 5Z'(x) + 6Z(x) = f(x), Z(0) = 0, Z'(0) = 0. Find the solution of y''(x) + 5y'(x) + 6y(x) = f(x), y(0) = a, y'(0) = b.