

# Paul Apisa

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CONTACT INFORMATION	Office 321, Van Vleck Hall, 480 Lincoln Dr, Madison, WI 53706 e-mail: paul.apisa@gmail.com	Phone: 937 469 2330 Citizenship: US
RESEARCH INTERESTS	Dynamics, geometry, Teichmüller theory	
ACADEMIC APPOINTMENTS	<b>University of Wisconsin - Madison, Department of Mathematics</b> <i>Assistant Professor</i> August 2022 - present	
	<b>University of Michigan, Department of Mathematics</b> <i>Donald J. Lewis Research Assistant Professor</i> August 2021 - August 2022 <i>NSF Postdoctoral Fellow</i> July 2020 - August 2021	
	<b>Yale University, Department of Mathematics</b> <i>Gibbs Assistant Professor</i> July 2019 - July 2020 <i>NSF Postdoctoral Fellow</i> July 2018 - June 2019	
EDUCATION	<b>The University of Chicago, Department of Mathematics</b> <i>Ph.D. - Thesis: Dynamics on the moduli space of translation surfaces</i> June 2018 <i>M.S.</i> September 2014 <b>The Ohio State University</b> <i>Bachelor of Science in Mathematics; with distinction and honors research distinction</i> June 2012	<b>Advisor: Alex Eskin</b>
PAPERS	<ol style="list-style-type: none"><li>High rank invariant subvarieties (with Alex Wright) <i>Ann. of Math. (2)</i> 198 (2023), no. 2, 657–726.</li><li><math>GL(2, \mathbb{R})</math> orbit closures in hyperelliptic components of strata. <i>Duke Math J.</i> (2018), Vol. 167, No. 4, 679–742.</li><li>Rank one orbit closures in <math>\mathcal{H}^{hyp}(g-1, g-1)</math>. <i>Geom. Funct. Anal. (GAFA)</i> 29 (2019), no. 6, 1617–1637.</li><li><math>GL(2, \mathbb{R})</math>-invariant measures in marked strata: Generic marked points, Earle-Kra for strata, and illumination. <i>Geom. Topol.</i> 24 (2020), no. 1, 373–408.</li><li>Exceptional directions for the Teichmüller geodesic flow and Hausdorff dimension (with Hamid al-Saqban, Alena Erchenko, Osama Khalil, Shahriar Mirzadeh, and Caglar Uyanik). <i>J. Eur. Math. Soc. (JEMS)</i> 23 (2021), no. 5, 1423–1476.</li><li>Marked points on translation surfaces (with Alex Wright). <i>Geom. Topol.</i> 25 (2021), no. 6, 2913–2961.</li><li>Divergent on average directions of Teichmüller geodesic flow (with Howard Masur). <i>J. Eur. Math. Soc. (JEMS)</i> 24 (2022), no. 3, 1007–1044.</li><li>Generalizations of the Eierlegende-Wollmilchsau (with Alex Wright) <i>Camb. J. Math.</i> 10 (2022), no. 4, 859–933.</li><li>Moduli spaces of complex affine and dilation surfaces (with Matt Bainbridge and Jane Wang) <i>J. Reine Angew. Math.</i> 796 (2023), 229–243.</li><li>Periodic points on the regular and double <math>n</math>-gon surfaces (with Rafael Saavedra and Christopher Zhang). <i>Geom. Dedicata</i> 216 (2022), no. 6, Paper No. 69, 17 pp.</li><li>A generalization of the Burnside basis theorem (with Benjamin Klopsch). <i>J. of Alg.</i> 400 (2014), 8–16.</li><li>Reconstructing orbit closures from their boundaries (with Alex Wright) <i>Mem. Amer. Math. Soc.</i>, accepted, available at <a href="https://arxiv.org/abs/2011.08807">arXiv:2011.08807</a>.</li><li>Periodic points in genus two: Holomorphic sections over Hilbert modular varieties, Teichmüller dynamics, and billiards. <i>preprint arXiv:1710.05505</i>.</li><li>Invariant subvarieties of minimal homological dimension, zero Lyapunov exponents, and monodromy <i>preprint arXiv:2103.02133</i>.</li></ol>	

PAPERS (CONT.)	15. Billiards in right triangles and orbit closures in genus zero strata <i>preprint arXiv:2110.07540.</i>
	16. Hurwitz-Hecke invariant subvarieties <i>preprint arXiv:2310.13776.</i>
	17. Algebraically primitive invariant subvarieties with quadratic field of definition (with David Aurlicino) <i>preprint arXiv:2404.10273.</i>
	18. Affine surfaces, monodromy, and the moduli space of regular connections (with Matt Bainbridge and Jane Wang) <i>to appear as a preprint (May 2024)</i>
AWARDS	2023 - present <b>NSF DMS Award No. 2304840</b> (\$250,000)
	2018 - present <b>NSF Mathematical Sciences Postdoctoral Research Fellowship</b>
	2013 - 2018 <b>NSF Graduate Research Fellowship</b>
	2012 - 2013 <b>McCormick Graduate Fellowship (University of Chicago)</b>
SEMINAR SERIES AND MINI-COURSES	Jan.-Feb. 2021 <b>Pacific Dynamics Winter 2021 Seminar Series</b> Five part seminar series given with Alex Wright.
	Sept. 2019 <b>Mini-course on the Moduli Space of Translation Surfaces at Symbolic Tools in Mathematics and their Application - Around Moduli Spaces</b> Joint program of Saarbrücken, Kaiserslautern and Aachen.
INVITED TALKS AND COLLOQUIA	Sept. 2024 <b>Oberwolfach - Riemann Surfaces: Random, Flat, and Hyperbolic Geometry</b>
	Apr. 2024 <b>Indiana University Dynamics Seminar</b>
	Mar. 2024 <b>Cornell Geometry and Topology Seminar</b>
	Oct. 2023 <b>Binghamton Geometry and Topology Seminar</b>
	<b>Notre Dame Geometry and Topology Seminar</b>
	May 2023 <b>Groups and Dynamics in Geometry Conference (Ascona)</b>
	Apr. 2023 <b>University of Chicago Geometry and Topology Seminar</b>
	Jan. 2023 <b>AMS Special Section on Geometry and Billiards</b>
	Oct. 2022 <b>Finiteness results for special subvarieties: Hodge theory, o-minimality, dynamics workshop</b> (Expository talk on affine invariant subvarieties)
	Sept. 2022 <b>Combinatorics, Dynamics and Geometry on Moduli Spaces</b>
	Mar. 2022 <b>Purdue University Colloquium</b>
	Feb. 2022 <b>Pennsylvania State University Colloquium</b>
	<b>Arizona State University Colloquium</b>
	Jan. 2022 <b>Northwestern University Colloquium</b>
	<b>Rice University Colloquium</b>
	<b>Michigan State University Colloquium</b>
	<b>Auburn University Colloquium</b>
	<b>Texas State University Colloquium</b>
	Dec. 2021 <b>University of Wisconsin Colloquium</b>
	<b>University Illinois at Chicago Colloquium</b>
	Nov. 2021 <b>University of Washington Colloquium</b>
	<b>Midwest Dynamics Conference</b>
	<b>University of Michigan Geometry Seminar</b>
	Oct. 2021 <b>Pennsylvania State University Dynamics Seminar</b>
	Sept. 2021 <b>Indiana University Dynamics Seminar</b>
	April 2021 <b>Midwest Dynamics Seminar</b>
	<b>Pennsylvania State University Dynamics Seminar</b>
	Sept. 2020 <b>BiSTRO - Billiards and Surfaces à la Teichmüller and Riemann, Online</b>
	May 2020 <b>Harvard Informal Geometry and Dynamics Seminar</b>
	<b>Nearly Carbon Neutral Geometry and Topology Conference</b>
	Oct. 2019 <b>Boston College Dynamics Seminar</b>
	Sept. 2019 <b>University of Maryland Dynamics Seminar</b>
	May 2019 <b>BIRS (Oaxaca) - Flat Surfaces and Dynamics on Moduli Spaces II Program</b>
	<b>Fudan University Seminar</b>
	April 2019 <b>Geometry and Topology at Brown and Yale (GATSBY) Seminar Pre-talk</b>
	<b>University of Chicago Dynamics Seminar</b>
	Dec. 2018 <b>California Institute of Technology - Geometry/Topology Seminar</b>
	Nov. 2018 <b>University of Michigan - Geometry Seminar</b>
	Oct. 2018 <b>Yale University - Group Actions and Dynamics Seminar</b>
	Sept. 2018 <b>Yale University - Geometry and Topology Seminar</b>
	<b>Oberwolfach - Workshop on Flat Surfaces and Algebraic Curves</b>

INVITED TALKS  
AND COLLOQUIA  
(CONT.)

- June 2018 **Teichmüller Dynamics, Mapping Class Groups and Applications**  
Institut Fourier, Grenoble
- April 2018 **AMS Sectionals - Vanderbilt University**  
Interactions between Geometry, Group Theory and Dynamics Special Section  
**Bloomington Geometry Workshop**
- Mar. 2018 **EPSRC Symposium on Geometry, Topology, and Dynamics in Low Dimensions**
- Aug. 2017 **The 13th William Rowan Hamilton Geometry and Topology Workshop**
- May 2017 **Chicago Action Now Dynamics Conference - Northwestern University**
- May 2016 **BIRS (Oaxaca) - Flat Surfaces and Dynamics on Moduli Spaces Program**
- April 2016 **Stanford University - Informal Geometry and Topology Seminar**
- Mar. 2016 **AMS Sectionals - University of Georgia**
- Mar. 2016 **University of Utah - Max Dehn Seminar**
- Sept. 2015 **Indiana University Bloomington - Geometry Seminar**
- July 2015 **Dynamics and Geometry in Teichmüller Space Conference at CIRM, Luminy**
- Mar. 2012 **AMS Sectionals - University of Kansas**

PHD STUDENTS

**Ruocheng Yang** (current)

RESEARCH  
EXPERIENCES FOR  
UNDERGRADUATES

- **Summer@ICERM 2021 REU Organizer (Summer 2021)**. Along with Diana Davis and Jane Wang, organized an REU focusing on computational aspects of flat surfaces. We ran nine projects for 19 students and organized a team of five other mentors.
- **Laboratory of Geometry Winter 2021 Project Organizer (Winter 2021)**. Along with Chaya Norton designed and ran a course directing a team of four undergraduates in a research problem on a computational question in the study of flat surfaces.
- **University of Michigan REU Organizer (Summer 2020)**. Along with Chaya Norton, Alex Wright, and Becca Winarski, ran an REU program for three students on Teichmüller dynamics and flat surfaces.
- **Cornell University REU Project Leader (Summer 2013)**. Along with Keith Dennis, mentored six students on a project on finite group theory.

MENTORING

- **RTG Seminar Organizer**. Organized a weekly dynamics seminar for graduate students covering topics such as Lyapunov exponents of the KZ cocycle (Fall 2023) and Patterson-Sullivan Theory (Spring 2024).
- **Margulis super-rigidity seminar organizer (Spring 2023)**. This was a learning seminar for graduate students.
- **Directed Reading Mentor (Spring 2014, Winter - Spring 2015; Fall 2017 - Spring 2018; Fall 2022 - Spring 2023; Spring 2024)**. Met weekly with undergraduate and graduate students and designed reading programs and problem sets for them to assist them in studying advanced topics of their choosing
- **Univ. of Michigan Postdoc-Grad Mentoring Program (Fall 2020)**. Mentored two graduate students.
- **Univ. of Chicago Big Sibling Mentoring Program (Winter 2017-Spring 2017)**. Mentored a lower-year graduate student and volunteered for a program in which younger students are allowed to observe, and ultimately teach, classes of older graduate students.

MATHEMATICAL  
OUTREACH

- **Assn. of Women in Mathematics Symposium Speaker (February 2017)**. Delivered a talk for a general audience on the work of Marina Ratner and Maryam Mirzakhani. The goal of the symposium was to increase awareness of the work of women in mathematics and to increase the participation of women in mathematics.
- **Math Circles (Fall 2020)**. Led a Math Circles, an enrichment activity for middle and high school students, on planar graphs and Euler characteristic.

SERVICE

- 2025 - Co-organizer of a conference in honor of Alex Eskin
- 2024 - NSF Panelist
- 2023 - Co-organizer of the Riemann Surfaces Session of the 8th Iberoamerican Congress on Geometry
- 2023-2024 - Undergraduate curriculum committee

- 2023 - Graduate admissions committee
- 2022-2024 - Co-organizer of the University of Wisconsin - Madison Putnam Club

TEACHING

**Course Instructor at the University of Wisconsin**

Winter 2024	Intro. to Ergodic Theory and Dynamics (graduate)	Math 758
Fall 2023	Modern Algebra 2	Math 542
Winter 2023	Linear Algebra	Math 341
Fall 2022	Linear Algebra	Math 341

**Course Instructor at the University of Michigan**

Winter 2022	Differential Geometry	Math 433
Fall 2021	Calculus	Math 115

**Course Instructor at Yale University**

Spring 2020	Introduction to Algebraic Topology 2 (Graduate) <i>This course covered vector bundles, K-theory, and the Atiyah-Singer index theorem.</i>	Math 545b
Fall 2019	Introduction to Algebraic Topology 1 (Graduate) <i>This course prepared PhD students for a qualifying exam on homotopy, homology, and cohomology.</i>	Math 544a

**Course Instructor at the University of Chicago**

2016 - 2017	Calculus	Math 151 - 153
2015 - 2016	Calculus, Multivariable Calculus, Linear Algebra	Math 153, Math 195-196

**College Fellow at the University of Chicago** - Led problem sessions and taught several classes.

Spring 2014	Partial Differential Equations	<i>Instructor:</i> Luis Silvestre
Winter 2014	Functional Analysis	<i>Instructor:</i> Panagiotis Souganidis
Fall 2013	Complex Analysis	<i>Instructor:</i> Sidney Webster

SEMINARS

All seminars met weekly.

(CO)-ORGANIZED

Fall 2022 - present	University of Wisconsin - Dynamics Seminar
Fall 2019 - Spring 2020	Yale Geometry & Topology Seminar
Fall 2018 - Spring 2019	Mirzakhani's Thesis Seminar