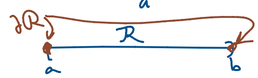
Green's Theorem

Fund. Thm. of Calculus: $\int_a^b f'(x)\,dx = f(b) - f(a)$



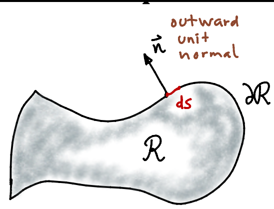
$\partial\mathcal{R} = \{a, b\}$

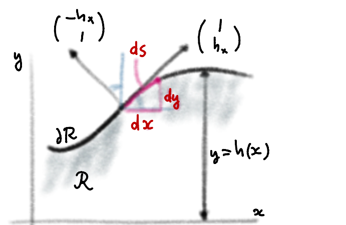
Partial integration = F Thm of Calc + Prod Rule:

$$\int_a^b f'(x)\,g(x)\,dx = \int_a^b (f(x)\,g(x))_x - f(x)\,g'(x)\,dx$$

$$= \Big[f(x)\,g(x)\Big]_a^b - \int_a^b f(x)\,g'(x)\,dx\,.$$

$\vec{n}$ and ds for plane domains





$$ds = \sqrt{(dx)^2 + (dy)^2} = \sqrt{1 + h_x^2}\;dx$$

$$\vec{n} = \frac{1}{\sqrt{1+h_x^2}}\begin{pmatrix}-h_x\\1\end{pmatrix}$$

$\vec{V} : \mathcal{R} \to \mathbb{R}^2$ $\quad \vec{V}(x,y) = \begin{pmatrix}V_1(x,y)\\V_2(x,y)\end{pmatrix}$ $\quad \operatorname{div}\vec{V} = \frac{\partial V_1}{\partial x_1}(x,y) + \frac{\partial V_2}{\partial x_2}(x,y)$

$D\vec{V}$ = all partials of all components of $\vec{V}$ $= \begin{bmatrix}\frac{\partial V_1}{\partial x_1} & \frac{\partial V_1}{\partial x_2}\\ \frac{\partial V_2}{\partial x_1} & \frac{\partial V_2}{\partial x_2}\end{bmatrix}$

$\operatorname{div}\vec{V}$ = Trace of $D\vec{V}$ = sum of diagonal elements of $D\vec{V}$.

Version 1 of Green's theorem, aka the Divergence Theorem

$$\iint_{\mathcal{R}} \operatorname{div}\vec{V}\;dx\,dy = \oint_{\partial\mathcal{R}} \vec{V}\cdot\vec{n}\;ds$$

Version 2

$$\iint_{\mathcal{R}} \left\{\frac{\partial V_1}{\partial x_1} + \frac{\partial V_2}{\partial x_2}\right\} dx_1\,dx_2 = \left(\oint_{\partial\mathcal{R}} V_1 dx_2 - V_2 dx_1\right)$$