MATH 561 - SECOND HOMEWORK ASSIGNMENT

due: October 2, 2003.

- 1. (a) Which point on the graph of $y = \ln x$ has the largest curvature? First make a guess from a decent drawing of the graph, then compute the answer.
 - (b) Same question for the the graph of $y = e^x$ (*Hint:* after doing (a), this should take about 10 seconds).
- 2. Consider the parametrized curve $\gamma(t) = \begin{pmatrix} t \\ t^2/2 \\ t^3/3 \end{pmatrix}$.
 - (a) Compute the curvature and torsion of γ at $\gamma(t_0)$ where t_0 is arbitrary.
 - (b) Let V_h be the plane through $\gamma(-h)$, $\gamma(0)$ and $\gamma(h)$. Find an equation for V_h in the form ax + by + cz = d (where a, b, c and d depend on h).
 - (c) Make a full-page drawing containing: The x, y, and z axes, the projections of the curve γ onto the xy plane and xz planes, the curve γ itself, and the plane V_h.
 - (d) Find the limit as $h \rightarrow 0$ of the plane V_h .
- 3. Let $\gamma : (a,b) \to \mathbf{R}^3$ be any parametrized curve, and assume that its curvature κ does not vanish.
 - (a) State the Frenet-Serret formulas for γ .
 - (b) Find a formula for $\frac{d^2 \mathbf{T}}{ds^2}$ in terms of \mathbf{T} , \mathbf{N} and \mathbf{B} .
 - (c) Show that $\mathbf{T} \cdot \frac{d^2 \mathbf{T}}{ds^2}$ is always negative.