Math 376 *Wednesday, March 30, MMV*

Second Midterm

Your Name:

1 Consider the surface parametrization $\vec{r} : \mathbb{R}^2 \to \mathbb{R}^3$ given by

$$\vec{r}(u,v) = \begin{pmatrix} u \\ uv \\ v^2 \end{pmatrix}.$$

(***i***) At which points is the parametrization $\vec{r}$ smooth ("regular")?

(***ii***) At which point(s) is the tangent plane to the surface parallel to the plane $4x + 2y + z = 0$?

Consider the surface $\mathcal{S} \stackrel{\text{def}}{=} \{\vec{r}(u,v) \mid 1 \leq u, v, \leq 2\}$, with $\vec{r}$ as above.

(***iii***) Express the area of $\mathcal{S}$ as a double integral of the type $\iint_{\mathcal{R}} f(u,v)\mathrm{d}u\mathrm{d}v$, where you explicitely state what $\mathcal{R}$ and $f(u,v)$ are.

2 Let $\mathcal{R}$ be the rectangle $[1,2] \times [1,2]$, and let $\mathcal{D}$ be the image of $\mathcal{R}$ under the map $F : \mathbb{R}^2 \to \mathbb{R}^2$ given by

$$F(u,v) = \begin{pmatrix} X(u,v) \\ Y(u,v) \end{pmatrix} = \begin{pmatrix} u/v \\ uv \end{pmatrix}.$$

(***i***) Sketch $\mathcal{D}$.

(***ii***) Compute the area of $\mathcal{D}$.

[*Problems 3 & 4 on the back of this sheet*]

SCORE • 1: 2: 3: 4: Total:

3 Let $\mathcal{R} \subset \mathbb{R}^3$ be the three dimensional solid given by

$$\mathcal{R} = \left\{(x, y, z) \mid x^2 + y^2 \leq R^2, 0 \leq z \leq x^2\right\}$$

in which $R > 0$ is a constant.

(***i***) Sketch $\mathcal{R}$.

(***ii***) The *moment of inertia about the z axis* of $\mathcal{R}$ is defined by

$$I_z = \iiint_{\mathcal{R}} \left(x^2 + y^2\right) \mathrm{d}x\mathrm{d}y\mathrm{d}z$$

Compute I_z by choosing convenient coordinates.

4 (***i***) Consider the vector fields

$$\vec{\boldsymbol{v}}\,(x, y, z) = -y\vec{\boldsymbol{i}} + x\vec{\boldsymbol{j}} + z\vec{\boldsymbol{k}}\,.$$

Use Stokes' Theorem to compute

$$\iint_{\alpha} \vec{\boldsymbol{v}} \cdot \vec{\boldsymbol{t}}\ \mathrm{d}s,$$

where α is the triangle whose corners are the origin, $(1, 0, 0)$ and $(1, 2, 0)$. (Choose your own orientation for α.)

(***ii***) Let $\mathcal{R} \subset \mathbb{R}^3$ be any bounded region with smooth boundary $\mathcal{S}$. Let $\vec{\boldsymbol{n}}$ denote the ***outward*** unit normal to $\mathcal{S}$. Consider the vector field

$$\vec{\boldsymbol{w}}\,(x, y, z) = x\,\vec{\boldsymbol{i}} + y^3\,\vec{\boldsymbol{j}} + z^5\,\vec{\boldsymbol{k}}\,.$$

Using Green's Theorem, decide if the following statement is ***True or False:*** The surface integral

$$\Phi \stackrel{\text{def}}{=} \iint_{\mathcal{S}} \vec{\boldsymbol{w}} \cdot \vec{\boldsymbol{n}}\ \mathrm{d}S$$

can never be negative.