Math 376 *Monday, February 21, MMV*

First Midterm

Your Name:

1 Let the curve α be the boundary of the region $\mathcal{D} = \{(x,y) : 0 \le x \le \pi, |y| \le \sin x\}$. Compute the integral

$$I = \oint_\alpha x\mathrm{d}y.$$

2 Let $\mathcal{D} \subset \mathbb{R}^2$ be the interior of a simple closed curve α, and let $\vec{\boldsymbol{n}}$ be the outward unit normal on α.

If $\vec{\boldsymbol{f}}(x,y) = \begin{pmatrix} x+\sin x \\ 2y + y\cos x \end{pmatrix}$ then show that there is an integer k such that

$$\oint_\alpha \vec{\boldsymbol{f}} \cdot \vec{\boldsymbol{n}}\, \mathrm{d}s = k \cdot \mathrm{area}(\mathcal{D}),$$

and find the value of k.

3 Let $\mathcal{T} \subset \mathbb{R}^2$ be the plane triangle with corners $(0,0)$, $(1,1)$ and $(3,1)$. Compute

$$J = \iint_{\mathcal{T}} (x^2+y^2)\,\mathrm{d}x\mathrm{d}y.$$

4 Let β be ***any*** curve in $\mathbb{R}^3$ connecting the origin to the point $(3,4,-1)$. Compute

$$\int_\beta x^2\mathrm{d}x + y\mathrm{d}y - 2z\mathrm{d}z.$$

SCORE • 1: 2: 3: 4: Total: