1 For which values of $\alpha \in \mathbb{R}$ are the vectors

$$u = \begin{pmatrix} \alpha+2 \\ \alpha+2 \\ 2 \end{pmatrix}, \qquad v = \begin{pmatrix} 2 \\ \alpha \\ 1 \end{pmatrix} \text{ and } \qquad w = \begin{pmatrix} 2 \\ \alpha \\ 2 \end{pmatrix}$$

linearly independent?

2 Let V be the vectorspace of all polynomials of degree 2 or less, i.e.

$$V = \left\{ a + bx + cx^2 \mid a, b, c \in \mathbb{R} \right\}.$$

Consider the linear transformation $T : V \to V$ defined by

$$(Tf)(x) = f''(x) + xf'(x) - 2f(x).$$

(***i***) Compute the matrix of T with respect to the basis $\{1, x, x^2\}$.

(***ii***) Is $T : V \to V$ injective?

(***iii***) Is $T : V \to V$ surjective?

3 Given are the matrices

$$A = \begin{pmatrix} 1 & -2 & 3 \\ 0 & 2 & 0 \\ 1 & 0 & 0 \end{pmatrix}; \qquad B = \begin{pmatrix} 0 & 3 & 1 \\ -1 & 2 & 0 \\ 4 & 0 & 0 \end{pmatrix}$$

(***i***) Compute the matrixproduct AB.

(***ii***) Show that A has an inverse without actually computing A^{-1}.

(***iii***) Compute the determinants $\det(AB)$, $\det(A^{-1}B)$.

(***iv***) Compute the determinant $\det(A - B)$.

4 Consider the function f defined by

$$f(t) = \begin{vmatrix} t & 3 & t & 0 \\ t & t & 0 & 1 \\ -1 & 0 & 3 & 0 \\ 0 & 2 & 0 & 1 \end{vmatrix}$$

The function $f(t)$ is a polynomial in t.

What is its degree, and compute the term with the highest degree.